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## A CRITIQUE OF INDEX NUMBERS OF THE PRICES OF STOCKS

In recent years index numbers of stock prices have gained general acceptance: they are regularly “carried” by the financial press; they are watched by bankers, investors, and speculators; they are put before railway commissions and courts as evidence; they are used in many ways by publicists and economists. This acceptance, however, is not the result of critical approval. Perhaps the good repute which index numbers of commodity prices at wholesale have fairly won after long discussion has disposed most “consumers of statistics” to trust index numbers as such. But apart from any special justification, there certainly prevails an amiable willingness to take upon faith plausible figures that fill a pressing want. And the stock index numbers have been published in the form that makes new figures most alluring—the paucity of explanations and warnings has encouraged readers to use or misuse the results without undergoing the mental toil of criticism or the moral strain of doubt. As for the cautious minority, they have been foiled by this same simplicity of presentation; they have been given few materials wherewith to determine the representative value of the original quotations, to judge the appropriateness of the methods used, or to compare the results of rival series.

To test the value of these popular figures is the aim of this paper. The method chosen is to put side by side several index

numbers of stocks, old and new, to compare the differences among them with the corresponding differences among commodity indexes, to trace these differences to their sources in dissimilarities of data and methods, and finally to show which among the numerous forms of stock indexes is best for general use.

This plan of starting with the familiar figures now in common use and of proceeding in an empirical fashion to inquire what, if anything, is the matter with them, has the merits of a direct attack upon the concrete problem of chief interest. But on the other hand, the discussion developed along this line becomes discursive rather than systematic. All the usual questions concerning the construction of index numbers are faced at some stage of the inquiry—the use of relative or actual prices, the choice of averages, the problem of weighting, etc.; but the order in which these questions are faced is a matter rather of convenience than of predestination. The whole performance, indeed, has the defects of an experimental as opposed to a mathematical way of finding and dealing with problems.

This procedure would scarcely be feasible unless a general acquaintance with index numbers could be taken for granted. But with a background of such knowledge fixed in the minds of readers, it should be possible to turn our eyes now toward one, now toward another feature of the foreground without becoming confused. For this foreground represents merely the application of devices that have proved their value for measuring the price fluctuations of commodities to the more difficult task of measuring the price fluctuations of stocks. The sole preliminaries called for are a brief discussion of fundamental differences between these two classes of goods and a still briefer reference to the uses for which stock indexes are made.<sup>1</sup>

#### I. THE FUNDAMENTAL DIFFERENCE BETWEEN STOCK AND COMMODITY INDEX NUMBERS

In several respects stock prices are more satisfactory data for statistical analysis than commodity prices. Stock dealings are more highly centralized and more thoroughly organized than deal-

<sup>1</sup>The writer's hearty thanks are due to Professor James A. Field for generous suggestions and criticisms, by which he has profited in revising the first draft of this paper.

ings in most commodities. The prices are reported with unexcelled fulness and accuracy. While the number of stocks for which frequent and regular quotations can be collected for considerable periods is less than the corresponding number of commodities at wholesale, it doubtless forms a larger proportion of the whole list dealt in. Once more, stocks are quoted in terms of a nominally uniform unit—the share with a par value of \$100, or some multiple that can readily be changed into the standard unit. Hence the actual prices can be compared, summed, and averaged with a facility lacking when one handles commodity prices. Concerning the authenticity and the representative character of stock quotations, in short, there are fewer doubts than haunt the mind of the field-worker in a commodity-price investigation.

It is when one begins to interpret these quotations that doubts become grave. First there is the familiar question: What does the share with a par value of \$100 really mean? Second, there is the assurance that whatever that unit in one corporation means this year, it will probably mean something different next year. Commodities are tangible substances, measured by physical units, and in making index numbers one rejects articles that are not substantially uniform in quality over long periods. Business enterprises, on the contrary, are essentially variable entities, and shares in them are subject to changes that affect the enterprises, and to other changes as well. The Pennsylvania Railroad, for example, is a remarkably stable corporation; yet its physical property, its security holdings, its leases, its indebtedness, its earnings and expenses, its financial affiliations, its relations to regulating commissions, and a hundred other matters that affect the market value of its shares, all vary constantly or intermittently. To cite only one crude gauge: the Pennsylvania system counted about 7,600 miles of railway in 1890 and about 11,800 miles in 1915. And in this changing property a share of common stock in 1890 represented ownership of one part in 2,451,354, whereas in 1915 one share represented ownership of one part in 9,985,314. Stocks, then, are variable fractions of variable wholes, and their prices fluctuate incessantly because of changes *in the thing quoted*, as well as for other reasons.

From such facts it is sometimes inferred that index numbers of stock prices have no valid use except for short-period comparisons; or that an index number covering decades is no better when it excludes than when it admits numerous substitutions of one stock for another. In any case, the argument runs, comparisons of stock prices in years far apart are comparisons of dissimilar goods: they are like comparisons of the prices of potatoes and silk in 1890 with the prices of pig iron and tea in 1915.

Such conclusions, however, are rash. The fact that stocks change as commodities do not, proves merely that stock index numbers must not be interpreted as meaning precisely what commodity indexes mean. It does not prove that stock indexes are meaningless, or that alterations in the list of securities included in them are unobjectionable. Business enterprises, indeed, are more like men than they are like commodities. Commodities are produced and consumed; then produced afresh in the old forms. Business enterprises have a continuous life; they undergo great changes of expansion, contraction, even reorganization, without losing their identity. And this continuity of business enterprises and of shares in them is a fact of great practical importance. The many individuals and corporations that hold stock in the same business enterprises for years at a time are deeply concerned with long-period changes in the prices of their securities. The like holds true with reference to the "investing public" as a unit, and to its security holdings as an aggregate. Even the wider public in its efforts to regulate corporation charges and corporation finance through governmental commissions needs to know the course followed by security prices in particular and in general. The fluctuations of New Haven stock between the early nineties and the present are not rated a matter of indifference; neither are the very different fluctuations of Pennsylvania stock, nor the still different fluctuations of Lackawanna. Nor is it unimportant to find out which type of fluctuations has been characteristic of American stocks at large.

Stock indexes, then, differ from commodity index numbers in that they show, not variations in the prices of unvarying goods, but variations in the prices of goods that maintain their identity

despite continual changes in quality. This difference enhances the difficulty both of making and of using them; but it does not destroy their logical legitimacy or their practical importance.

With the many other differences between stocks and commodities we have slight present concern. Were it our task to explain the facts which index numbers reveal we should have to go at length into the peculiarities presented by the demand for, and the supply of, stocks. But in measuring given phenomena it is seldom necessary to know their causes in advance. On the contrary, measuring is generally a most important preliminary in conducting a search for causes, and so we should find it in this case, were we trying to account for the course of the stock market. What we do need to be sure of in advance is that our measures shall have meaning and use when they are made. Doubts on the first head having been set aside, we turn to the second.

## II. THE USES OF STOCK INDEX NUMBERS

An index number is a statistical device made to serve certain ends. Hence the logical first step toward an evaluation of any such series is to define precisely the end which the finished results are to serve. That done, one has a criterion by which to judge the merits and defects of the series already in use and by which to guide his own efforts in making new series.

The trouble with this seemingly promising lead is that stock index numbers are put to so many and such varied uses as to give little help in defining what is wanted. An economist may seek to measure changes in the purchasing power of money over stocks, a speculator may wish to forecast the probable future course of the market, a public commission may be interested in the terms on which corporations can raise new capital, a publicist may investigate the claim that government regulation has brought loss upon investors, a financial historian may wish to mark off periods of expansion and contraction, a trustee may inquire whether the fluctuations of his security holdings have compared favorably with the average course of the market, an insurance company may seek light on the probable future of interest rates, a student may wish to compare stock fluctuations with the price fluctuations of

commodities at wholesale or retail, of labor, of bonds, of farm lands, of securities in other countries, etc. Now, each one of these people will have use for a stock index. But the more carefully these various uses are analyzed, the clearer it becomes that their requirements differ. The character and the number of stocks to be included, the frequency of the quotations needed, the period of time covered, whether actual or relative prices should be used, the desirability of making subgroups and their basis, the kind of average appropriate, the necessity of considering deviations from the mean, whether weights should be introduced and if so what is the proper criterion of "importance"—these and the other points of technique that arise in making an index number would not all be decided precisely alike in any two of the cases suggested, did uses strictly dictate methods—as logically they should.

Ideally, every distinct use should have a distinct index number especially designed for it. Practically, however, cases are few when the consumer of statistics has the technical skill and can spend the time and money to make a series exactly answering his needs. What happens is that he uses for his special purposes one of the series published by others—more often than not without realizing that the figures in question are in certain respects ill adapted to his needs. Frequently the user does not even hit upon that one among the published series which is least unsuited to his case. And this situation promises to change but slowly. Probably the published series will long continue to be used as "general-purpose" index numbers. And a "general-purpose" index number is too indefinite a conception to guide one surely through the maze of choices that are involved in making a new series or in ranking old ones.

Under these confusing circumstances, what can we attempt with any prospect of success? We cannot discuss the merits of stock index numbers at large with reference to their uses, because these uses and their several requirements are so multifarious. Our best hope seems to lie in reversing the problem. That is, we can analyze stock index numbers, old and new, to find of what

materials and by what methods they are made. Then we can discuss their uses with reference to their construction. Finally, we can determine what fluctuations in the prices of stocks can be measured most accurately and by what means. The index number which stands first in this test will have special claims to acceptance, except for uses which require some radically different series, less accurate though it be.<sup>1</sup>

### III. A COMPARISON OF DIFFERENT INDEX NUMBERS OF THE PRICES OF AMERICAN STOCKS IN 1890-1915

Tables I, II, and III place side by side several index numbers of stocks in form proper for comparison. The first table shows annual averages of actual prices in dollars and cents; the second shows percentages of rise or fall in price from one year to the next; the third shows relative prices on the base: average actual prices in 1890-99=100. A few words of explanation concerning the different series will make their meaning clear.

When I began studying business cycles, I needed a measure of the fluctuations in the prices of stocks, comparable with the existing index numbers of commodity and labor prices. The most comprehensive tables of the latter sorts were those published by the United States Bureau of Labor Statistics. At that time the Bureau's index numbers were simple arithmetic means of relative prices computed on the base: average actual prices in 1890-99=100.0. Accordingly I adapted the new index number of stock prices to the Bureau's model. This series was brought down to date in the February, 1916, issue of the *Journal of Political Economy*, and is reproduced in Table III, column 2. It includes the common stocks of thirty-five railways and five express, steamship, and telegraph companies. Two figures—averages of the highest and lowest relative prices—have been published for each month, quarter, and year in 1890-1915; but in this paper only the annual averages are used—arithmetic means made from the twenty-four

<sup>1</sup> Compare the corresponding section in "The Making and Using of Index Numbers," *Bulletin of the United States Bureau of Labor Statistics*, Whole No. 173.



TABLE I

EIGHT INDEX NUMBERS OF THE AVERAGE ACTUAL PRICES OF STOCKS ON THE NEW YORK STOCK EXCHANGE. BY YEARS: 1890-1915  
(For explanations of the several series, see the accompanying text)

FORTY TRANSPORTATION STOCKS							
Simple Averages			Weighted Averages			Twenty RAILWAY STOCKS; <i>Wall Street</i> <i>Journal</i> Series	All Stocks SOLD ON THE NEW YORK STOCK EXCHANGE
Geometric Means	Medians	Arithmetic Means	Arithmetic Means Weighted by				
			Stock Outstanding (4)	Earnings (5)	Number of Shares Sold (6)		
1890.....	\$40.68	\$61.16	\$70.47	\$76.37	\$54.96	\$65.62*	\$60.20
1891.....	38.98	57.87	67.16	73.13	49.25	62.82*	57.10
1892.....	43.18	62.18	71.75	78.21	54.69	71.32*	63.50
1893.....	33.28	52.71	60.95	67.03	40.24	58.45*	60.30
1894.....	28.64	48.55	55.35	61.12	32.70	52.67*	64.20
1895.....	20.83	49.28	56.22	61.75	32.67	53.79*	60.30
1896.....	26.91	46.36	53.70	59.26	31.39	50.38*	65.20
1897.....	28.46	49.06	57.03	62.92	35.79	56.25	67.00
1898.....	31.85	54.36	63.67	70.39	42.68	65.29	72.70
1899.....	44.04	66.48	76.49	84.44	55.77	82.11	78.60
1900.....	45.26	67.67	77.55	85.80	58.11	79.60	69.20
1901.....	70.13	90.69	100.11	110.13	88.09	107.83	79.00
1902.....	82.86	105.48	111.85	125.29	99.38	119.36	79.90
1903.....	67.28	89.65	95.22	107.13	83.62	102.84	73.20
1904.....	64.84	89.44	94.82	107.10	86.00	102.38	69.90
1905.....	81.59	108.60	115.77	129.76	113.16	125.33	87.30
1906.....	86.86	117.42	122.15	137.84	126.97	131.24	94.20
1907.....	65.27	95.41	101.35	114.86	102.69	103.28	85.80
1908.....	62.90	94.78	100.39	114.66	106.44	102.15	86.60
1909.....	84.85	120.10	125.09	142.12	136.66	125.47	97.50
1910.....	75.10	108.77	116.71	130.70	126.85	117.21	96.20

1911.....	71.05	95.30	105.80	115.42	128.10	124.96	117.53	95.80
1912.....	68.01	96.45	106.83	116.87	129.45	124.97	119.57	97.70
1913.....	54.07	78.35	89.33	101.37	110.86	112.88	107.05	90.20
1914.....	43.95	65.35	79.44	90.07	98.85	102.23	97.35†	93.20‡
1915.....	43.53	67.90	80.56	90.72	99.69	100.51	87.01	85.10
Averages								
1890-1899.....	34.59	31.72	54.80	63.28	69.46	43.01	61.87	64.91
1900-1909.....	71.18	78.90	97.92	104.43	117.47	100.11	109.95	82.26
1910-1915.....	59.39	83.59	95.12	105.19	116.28	115.40	107.72	94.03
1890-1915.....	54.39	61.82	80.69	88.77	98.73	81.68	90.95	78.30

\* Computed from monthly lowest and highest prices of stocks in the *Wall Street Journal's* list as constituted in 1897.

† Lowest prices of July used for August to November, inclusive, when the Stock Exchange was closed.

‡ No transactions July 31 to December 11, inclusive.

TABLE II

TEN INDEX NUMBERS SHOWING THE PERCENTAGES OF RISE (+) OR FALL (—) IN THE PRICES OF STOCKS FROM ONE YEAR TO THE NEXT ON THE NEW YORK STOCK EXCHANGE. BY YEARS: 1890-91 TO 1914-15  
(For explanations of the several series, see the accompanying text)

FORTY TRANSPORTATION STOCKS									
Series Made from the Percentages of Rise or Fall in the Average Yearly Prices of Each Stock				Series Made from the Average Actual Prices Shown in Table I				Twenty Railway Stocks; <i>Wall Street Journal</i> Series from Table I	All Stocks Sold on the New York Stock Exchange from Table I
Simple Averages				Weighted Averages				(9)	(10)
Medians (1)	Arithmetic Means (2)	Geometric Means (3)	Medians (4)	Arithmetic Means (5)	Stock Outstanding (6)	Earnings (7)	Number of Shares Sold (8)		
Per Cent	Per Cent	Per Cent	Per Cent	Per Cent	Per Cent	Per Cent	Per Cent	Per Cent	Per Cent
1890-91.....	- 3.3	- 2.7	-11.1	- 5.4	- 4.7	- 4.2	-10.4	- 4.3	- 5.1
1891-92.....	+ 6.9	+13.7	+27.3	+ 7.4	+ 6.8	+ 6.9	+11.0	+13.5	+11.2
1892-93.....	-16.8	-21.4	-41.4	-15.2	-15.0	-14.3	-26.4	-18.0	- 5.0
1893-94.....	-10.9	-12.1	-26.4	- 7.9	- 9.2	- 8.8	-18.7	- 9.9	+ 0.5
1894-95.....	+ 4.3	+ 7.0	+24.0	+ 1.5	+ 1.6	+ 1.0	- 0.1	+ 2.1	- 6.1
1895-96.....	-10.5	- 8.1	-17.1	- 5.9	- 4.5	- 4.0	- 3.9	- 6.3	- 8.1
1896-97.....	+ 6.2	+10.4	+17.6	+ 5.8	+ 6.2	+ 6.2	+14.0	+11.6	+ 2.8
1897-98.....	+ 9.7	+14.5	+35.2	+10.8	+11.6	+11.9	+19.2	+16.1	+ 8.5
1898-99.....	+24.5	+49.7	+53.6	+22.3	+20.1	+20.0	+30.7	+25.7	+ 8.1
1899-1900.....	+ 0.2	+ 4.1	+ 5.9	+ 1.8	+ 1.4	+ 1.6	+ 4.2	- 3.1	-12.0
1900-1.....	+43.3	+61.7	+54.9	+34.0	+29.1	+28.3	+51.6	+35.5	+14.2
1901-2.....	+19.3	+19.3	+19.4	+16.3	+11.7	+13.8	+12.8	+10.7	+ 1.1
1902-3.....	-17.0	-18.1	-19.0	-15.0	-14.9	-14.6	-15.9	-13.9	- 8.4
1903-4.....	- 2.1	- 3.1	- 3.8	- 0.2	- 0.4	± 0.0	+ 2.8	- 0.5	- 4.5
1904-5.....	+24.3	+28.0	+26.0	+21.4	+22.1	+21.2	+31.6	+22.4	+24.9
1905-6.....	+ 3.4	+ 7.8	+ 3.5	+ 8.1	+ 5.5	+ 6.2	+12.2	+ 4.7	+ 7.9

1906-7.....	-22.4	-23.8	-24.8	-20.0	-18.7	-17.0	-16.7	-19.3	-21.3	-8.9
1907-8.....	-3.4	-2.9	-3.6	-3.0	-0.7	-0.9	-0.2	+3.7	-1.1	+0.9
1908-9.....	+30.3	+37.1	+34.9	+27.5	+26.7	+24.6	+23.9	+28.4	+22.8	+12.6
1909-10.....	-7.0	-10.0	-11.5	+0.5	-9.4	-6.7	-8.0	-7.2	-6.6	-1.3
1910-11.....	-5.1	-4.8	-5.4	-3.0	-2.7	-1.1	-2.0	-1.5	+0.3	-0.4
1911-12.....	-2.1	-2.3	-4.3	+1.2	+1.0	+1.3	+1.1	+0.0	+1.7	+2.0
1912-13.....	-17.2	-18.8	-19.6	-18.8	-16.4	-13.3	-14.4	-9.7	-10.0	-1.5
1913-14.....	-14.2	-17.5	-19.6	-16.6	-11.1	-11.1	-10.8	-9.4	-9.6	-3.1
1914-15.....	-0.4	+1.2	-1.0	+3.9	+1.4	+0.7	+0.9	-1.8	-10.6	-8.7
Averages										
1890-1900....	9.3	14.4	12.5	26.0	8.4	8.1	7.9	13.9	11.1	7.3
1900-1910....	16.9	21.2	20.3	17.6	15.1	13.3	13.3	18.6	14.0	8.5
1910-1915....	7.8	8.9	10.0	8.7	6.5	5.5	5.8	4.5	6.4	3.1
1890-1915....	12.1	16.0	15.1	19.2	10.7	9.7	9.6	13.9	11.3	7.0

TABLE III

ELEVEN INDEX NUMBERS SHOWING THE RELATIVE PRICES OF STOCKS ON THE BASIS: AVERAGE ACTUAL PRICES IN 1890-99=100.  
 BY YEARS: 1890-1915

(For explanations of the several series, see the accompanying text)

FORTY TRANSPORTATION STOCKS										
Series Made from the Relative Prices of Each Stock			Series Made from the Average Actual Prices Shown in Table I					FORTY COMMON AND TEN PREFERRED STOCKS; ARITHMETIC MEANS OF RELATIVE PRICES	TENTY RAILWAY STOCKS; <i>Wall Street Journal</i> SERIES FROM TABLE I	ALL STOCKS SOLD ON THE NEW YORK STOCK EXCHANGE FROM TABLE I
Simple Averages			Weighted Averages			Arithmetic Means Weighted by				
Medians (1)	Arithmetic Means (2)	Geometric Means (3)	Medians (4)	Arithmetic Means (5)	Stock Outstanding (6)	Earnings (7)	Number of Shares Sold (8)			
1890.....	110	118	127	112	111	110	128	118	106	93
1891.....	108	113	113	106	106	105	115	111	102	88
1892.....	115	125	143	113	113	113	127	121	115	98
1893.....	95	96	84	96	96	97	94	92	94	93
1894.....	88	83	62	89	87	88	76	82	85	99
1895.....	89	86	77	90	89	89	76	86	87	93
1896.....	82	78	64	85	85	85	73	79	81	100
1897.....	89	82	75	90	90	91	83	86	91	103
1898.....	93	92	101	99	101	101	99	97	106	112
1899.....	117	127	155	121	121	122	130	129	133	121
1900.....	125	131	164	123	123	124	135	134	129	107
1901.....	189	203	242	165	158	159	205	204	174	122
1902.....	220	240	289	193	177	180	231	238	193	123
1903.....	184	195	234	164	159	154	194	193	166	113
1904.....	172	187	225	163	150	154	200	185	165	108
1905.....	241	236	284	198	183	187	263	236	203	134
1906.....	238	251	294	214	193	198	295	250	212	145

1907.....	179	204	189	235	174	160	165	239	191	167	132
1908.....	170	201	182	228	173	159	165	248	187	165	133
1909.....	249	277	245	291	219	198	205	318	255	203	150
1910.....	210	254	217	310	108	184	188	205	233	180	148
1911.....	188	248	205	300	193	182	184	291	226	190	148
1912.....	185	243	197	304	195	185	186	291	219	193	151
1913.....	125	206	158	247	163	160	160	262	186	174	148
1914.....	112	178	127	206	145	142	142	238	159	157	144
1915.....	110	180	126	214	147	143	144	234	161	141	131
Averages											
1890-1899.....	99	100	100	100	100	100	100	100	100	100	100
1900-1909.....	197	219	206	249	179	165	169	233	207	178	127
1910-1915.....	153	218	172	264	174	166	167	268	197	174	145
1890-1915.... Net Rise (+) or Fall (-)	149	173	157	195	147	140	142	190	164	147	121
1890-1896....	- 28	- 44	- 40	- 63	- 27	- 26	- 25	- 55	- 39	- 25	7
1896-1909....	+ 167	+ 200	+ 167	+ 227	+ 134	+ 113	+ 120	+ 245	+ 176	+ 122	+ 50
1909-1915....	- 139	- 97	- 119	- 77	- 72	- 55	- 61	- 84	- 94	- 62	- 19
1890-1915.... Difference be- tween highest and lowest....	± 0	+ 59	+ 8	+ 87	+ 35	+ 32	+ 34	+ 106	+ 43	+ 35	+ 38
Average change from one year to the next ...	167	200	173	248	134	113	120	245	176	131	63
	23	24	23	30	15	13	13	23	22	16	8

NOTE.—The entries italicized in each series are the minimum and maximum.

monthly quotations (twelve high and twelve low) for each stock each year.<sup>†</sup>

In recent years, I have utilized the large body of data collected for this series to make twenty-one new index numbers. Of the new series, six are annual averages of actual prices made by first striking the arithmetic means of the highest and lowest monthly quotations to get the average yearly prices of each of the forty stocks, and then averaging these forty yearly figures in different ways. The results are shown in the first six columns of

<sup>†</sup> The list of stocks included in this series and certain facts concerning each are shown below in Tables VIII-X.

The source relied upon for the original data is the table of "Prices of Stocks at the New York Stock Exchange" published annually in the *Financial Review*. Two exceptions are to be noted: (1) Pennsylvania Railroad stock was not regularly quoted on the New York market until September, 1897. For earlier dates prices on the Philadelphia Exchange are used. (2) For October and November, 1914, when the New York market was closed, quotations are based on the prices given by brokers' circulars as digested by the *Financial Review*. The August and September figures in that year are made from the prices (largely bid and asked) of July 30. The quotations are prices on 100-share lots, and on par values of \$100. Two such prices are used for each stock each month—namely the highest and the lowest recorded. In reducing these monthly figures to annual averages no attempt was made to supply missing quotations in earlier years—if no transactions occurred in a certain stock in one month of a year, its average price was made by casting up the 22 quotations for the remaining months and dividing by 22. Since 1911, however, such gaps in the data have been filled in by the use of bid and asked prices or by interpolation. No preferred stocks are included.

In the original investigation all the railway stocks were selected that had approximately complete and regular price records for the years 1890-1909. Several important lines, such as the Burlington, Lake Shore, Michigan Central, and Northern Pacific, were omitted because quotations were scanty or altogether lacking for one or more years. Other lines, like the Rock Island and the Alton, have undergone reorganizations which broke the continuity of their quotations. A similar break has resulted in the case of the Pullman Company, the Adams Express Company, etc., from the issue of large stock dividends. But stock dividends which did not force prices suddenly to a lower level and the payment of stock assessments which did not raise prices suddenly to a higher level were disregarded. If all stocks affected in any degree by such changes had been rejected the list remaining would have been short indeed. In railways undergoing reorganization, the prices of voting-trust certificates have frequently been used in lieu of the prices of shares. In no case, however, have the original quotations accepted been altered in any way to allow for "disturbing factors." Since the publication of the tables was begun in 1910, no changes have been made in the list of stocks.

For the full monthly and quarterly results see the *Journal of Political Economy*, XVIII, 345-80; XXI, 500-522; XXIV, 126-157.

Table I. Twelve more series are made by converting these same averages of actual prices into *relative* prices on two bases: first, prices in the preceding year = 100 (Table II, columns 3-8); second, average prices in 1890-99 = 100 (Table III, columns 3-8). The three remaining series made from my own collection of data (Table II, columns 1 and 2, Table III, column 1) are new averages of relative prices.

My reason for working the same data up in so many different ways was partly to find what effects are produced by various methods of computation, but mainly to secure series adapted to special uses. While my first stock index was appropriate for comparison with the other figures I had to use, it was open to criticism on technical grounds. The familiar objections to haphazard weighting, to arithmetic means, and to fixed-base relatives apply to index numbers of stock prices as well as to commodity indexes. But it was not until Dr. Royal Meeker determined to change the Bureau's commodity index numbers to weighted aggregates of actual prices, reduced to relatives on the last-completed-year base, that I had weighted aggregates (and averages) of actual prices made from my stock quotations. Three systems of weights were used: (1) average amounts of stock outstanding, (2) average earnings, (3) average number of shares sold, the averages in each case covering the twenty-five years 1890-1914. The results appear in Table I, columns 4, 5, and 6, while the weights are presented and their uses discussed in Section X, below.

Meanwhile a desire to compare my own figures with those published by the *Wall Street Journal* had led me to make the unweighted arithmetic means of actual prices (Table I, column 3). The original *Wall Street Journal* figures likewise had to be recast for the comparison. The long-period table published at intervals in that *Journal* shows the highest and lowest average prices of twenty railway stocks each month since 1897, except for the period in 1914 when the Stock Exchange was closed. I have (1) interpolated figures for these missing months on the assumption that stocks were "carried" at the prices of July 30, (2) reduced the monthly figures to annual averages, and (3) carried back the series from 1897 to 1890. It is to be noted that the list of stocks used has



undergone important changes since the series was begun.<sup>1</sup> The new version of this series appears in column 7 of Table I.

The medians of actual prices (column 2, Table I) are merely a by-product of a table made in studying the dispersion of the quotations (see Table XI below). The corresponding geometric means (column 1) were made less for themselves than as raw materials for striking a true average of ratios of change in the prices of stocks—of which more later. Finally, column 8 in this table was copied from the *Financial Review* for 1915, p. 51. This *Review* reports (1) the approximate actual values of all the stocks sold each year on the New York Stock Exchange, (2) the total par value of these stocks, and (3) the total number of shares sold. Experiment with the figures shows that the “average prices” here reproduced are computed by dividing the first of these values by the second. Of course the resulting series is a weighted average, and

<sup>1</sup> The following schedule shows the extent of these changes:

STOCKS INCLUDED IN THE *WALL STREET JOURNAL'S* AVERAGES OF TWENTY  
RAILWAY STOCKS

Included Both in 1897 and in 1915	Included in 1897 but Not in 1915	Included in 1915 but Not in 1897
1. Atchison 2. Chesapeake & Ohio 3. Erie 4. Louisville & Nashville 5. New York Central 6. Reading 7. St. Paul 8. Northern Pacific 9. Southern Railway	1. Lake Shore 2. Burlington 3. Rock Island 4. Central Railroad of New Jersey 5. Cleveland, Cincinnati, Chicago & St. Louis 6. Missouri Pacific 7. Chicago & Northwestern 8. Wabash, preferred 9. Missouri, Kansas & Texas, preferred 10. New York, Susquehanna & Western, preferred 11. Manhattan Elevated	1. Baltimore & Ohio 2. Canadian Pacific 3. Delaware & Hudson 4. Illinois Central 5. Kansas City Southern 6. Lehigh Valley 7. New Haven 8. Norfolk & Western 9. Pennsylvania 10. Southern Pacific 11. Union Pacific

For the backward extension of this series I used the monthly highest and lowest prices of the stocks in the list as it stood in 1897. One substitution was necessary because of lack of quotations: namely, Iowa Central preferred for Southern Railway preferred in January, 1890, to June, 1893. To reduce the effect of this change I multiplied the Iowa Central figures by 1.0453—the factor which makes the quotation for July, 1893, equal the price of Southern when it was first quoted.

Highest and lowest quotations for each stock each month, such as I took from the *Financial Review*, seldom yield precisely the same results as the highest and lowest daily averages for twenty stocks. As was to be expected, my new figures for the *Wall Street Journal* series in 1897 were one a little higher and the other a little lower than the *Journal's* figures: my average was 1.5 per cent higher. Hence I divided my new figures for 1890-96 by 1.015 to make as good a splice as possible.

My hearty thanks are due to Mr. Lockwood Barr, the editor of the *Wall Street Journal*, for information about the early history of his series.

one in which the weights change radically from year to year—that is, the larger the par value of the transactions in a given stock in a given year, the greater is the influence that stock exercises upon the average.

One great advantage of such averages of actual prices as are given in Table I is that they can be turned at will into relatives on any base desired.<sup>1</sup> To summarize: the eight indexes of the first table yield eight of the ten series of relatives in the second table, and eight of the eleven series of relatives in the third. The other series in Table II (columns 1 and 2) are made directly from the relative prices of each of the forty stocks, computed on the base: prices in the year before = 100. The same is true of columns 1 and 2 in Table III, except that the base in this case is average prices in the decade 1890–99. The one series left, column 9 in Table III, is similar in construction but different in materials; here relatives of ten preferred stocks are averaged with the relatives of forty common stocks employed in making column 2.<sup>2</sup>

#### IV. THE DIVERGENCES AMONG INDEX NUMBERS OF STOCKS AND AMONG INDEX NUMBERS OF COMMODITIES

The first test to be tried upon these index numbers of stock prices is the test of consistency. Elsewhere I have argued that we are justified in trusting our index numbers of commodity prices at wholesale, because series made by different men, from dissimilar data and by unlike methods, yield substantially similar results, and because the moderate discrepancies that do appear among them are consistent with what we know about the dissimilarities of fluctuation characteristic of different parts of the

<sup>1</sup> On the other hand, the index numbers made from averages of relative prices can be regarded as averages of actual prices made with a peculiar scheme of weights. (Compare F. R. Macaulay, "Index Numbers for Retail Prices," *American Economic Review*, V [December, 1915], 928, 929.) In that light, it is legitimate to turn them also into relatives on any base desired. The real objection to this procedure is that it becomes difficult to state clearly what such series mean after they have been shifted to a new base. The relation between relative prices and actual prices in this respect will be discussed more fully in section X below.

<sup>2</sup> This series was first published for 1890–1909 in this *Journal*, XVIII (July, 1910), 524. For the index numbers of preferred stocks see Table XX, column 2, below.

system of prices.<sup>1</sup> Can these reassuring conclusions be extended to index numbers of stock prices?

Certainly the series of actual prices (Table I) diverge notably from each other, even with respect to their broadest results—the twenty-six-year averages. The simple arithmetic means are 48 per cent higher than the corresponding geometric means, and 31 per cent higher than the medians made from precisely the same data. Weighting makes smaller but still considerable differences in results, the highest weighted series running 21 per cent above the lowest weighted series. While the latter series (No. 6) gives grand averages not 1.3 per cent greater than the corresponding simple arithmetic means (No. 3), that is the net result of large minus differences in the first half of the period offsetting large plus differences in the second half. Differences in the samples used make differences in the results, which though smaller again are still considerable: the twenty-six-year average of the *Wall Street Journal* series for twenty stocks is 13 per cent higher than the averages of the corresponding series for forty stocks (No. 3). Finally the combination of dissimilar samples and dissimilar weights yields differences in the averages which run from 4 to 26 per cent of the smaller series (No. 8 compared with Nos. 4, 5, and 6).

These differences, however, are a matter of small concern; for index numbers are intended to measure, not the average prices of stocks as such, but either the fluctuations of these average prices or average fluctuations in prices. Price fluctuations in the first of these senses can be followed more readily when the actual prices of Table I are turned into the relative prices of Tables II and III. Besides, the latter tables give additional series showing average fluctuations of prices, and also allow us to set commodity index numbers against our figures for stocks.

Do the stock indexes in the form of relative prices harmonize well with each other, as do the commodity indexes?

1. Concerning the *direction* in which prices move, stock indexes agree rather better than do commodity indexes. The eleven series of Table III all rise together or fall together in fourteen years;

<sup>1</sup> See "The Making and Using of Index Numbers," *Bulletin of the U.S. Bureau of Labor Statistics*, No. 173, especially pp. 112, 113.

they disagree in eleven years.<sup>1</sup> Two of the series, those made respectively from the medians of actual prices and from the prices of all stocks sold on the Exchange (Nos. 4 and 11), are most erratic as compared with the others. Leaving them out, we have agreement in seventeen years, disagreement in eight. A similar comparison of seven commodity indexes for 1890-1914 shows agreement in twelve years and disagreement in twelve.<sup>2</sup>

2. Stock indexes are more variable than commodity indexes in the sense that they show greater changes from one year to the next. When these changes are computed as percentages of prices in the preceding year (Table II), the ten series for stocks show average changes for twenty-five years ranging between 7 and 19 per cent. But the extremes are supplied by the two series found to be most erratic in the preceding comparison. Dropping them again, we have average variations of 10-16 per cent (Nos. 7 and 2). A similar computation for four commodity indexes on the preceding-year base yields the following averages: 3.6 per cent; 3.7 per cent; 4.6 per cent; 5.1 per cent.<sup>3</sup>

If these year-to-year changes are expressed as percentages of prices in 1890-99 (Table III), the figures for stocks run still higher: the extremes are now 8 and 30 per cent, or, after throwing out the two erratic series (Nos. 4 and 11 in this table), 13 and 24 per cent. Similar figures for eight commodity indexes show average changes varying from 4.0 to 7.9 per cent. Hence the most variable commodity series (Gibson's index of food prices) barely laps the steadiest of all the stock indexes, No. 11.<sup>4</sup>

<sup>1</sup> In a few cases where the change is so slight as not to appear on the coarse scale of relative prices used in Table III, reference is made to the actual prices of Table I to determine whether the trend was upward or downward.

<sup>2</sup> See *Bulletin of the U.S. Bureau of Labor Statistics*, No. 173, pp. 98, 99, for the results in 1890-1913. In 1914 the testimony is particularly conflicting.

<sup>3</sup> The series in question are, respectively, arithmetic means of the Bureau of Labor Statistics percentages of change from year to year (1890-1913), the Bureau's weighted aggregates of actual prices (1890-1914), Dun's index number (1890-1913), and Bradstreet's index (1890-1913). See the Bureau's *Bulletins* No. 173, p. 108, and No. 181, p. 19. I do not use the Bureau's own unweighted series of this type, because it is not computed in the proper way. Compare the explanation of method, p. 17 of *Bulletin* No. 181, with the criticism on pp. 39-42 of *Bulletin* No. 173.

<sup>4</sup> See *Bulletin* No. 173, p. 98. The corresponding figure for the Bureau's new weighted index is 4.0 per cent (*Bulletin* No. 181, p. 16).

Granted the greater variability of stock indexes in this sense, however, it cannot be said that the differences of variability among them (once more omitting Nos. 4 and 11) are wider in proportion to their magnitude than the corresponding differences among the commodity indexes.

3. Long-period comparisons give a result less favorable to the consistency of stock indexes. Net changes between 1890 and 1915, reckoned on the 1890-99 base, are made out to be zero by one series (Table III, No. 1), while another series (No. 8) shows a rise of 106 per cent. Eight commodity indexes in this case all concur in showing that prices rose between 1890 and 1913, and they make this rise come within the moderate limits of 17-35 per cent.<sup>1</sup> This marked contrast between the comparative harmony of commodity indexes and the wide divergences of stock indexes over considerable periods of time is doubtless due in part to the fundamental difference between the two classes of goods pointed out in Section I.

4. Averages of the yearly differences among the several indexes make closer comparison possible, but require a more elaborate scheme of presentation. This is provided by Tables IV and V, based respectively upon Tables II and III. In both tables, corresponding figures concerning commodity indexes are added for comparison.

In the series on the preceding-year base (Table IV) three cases of agreement appear among stock indexes (Nos. 6 and 7, 5 and 7, 5 and 6) closer than any pair of commodity indexes can show. Further, there are three other cases of agreement between two stock series closer than that between the most divergent pair of commodity indexes (Nos. 2 and 3, 1 and 5, 6 and 9). But on the other hand most of the forty-five comparisons (each of which is entered twice in the table) show differences much wider than any found between commodity series. Even if we drop once more the most erratic couple of stock indexes (Nos. 4 and 10) we find that the remaining eight differ among themselves by margins that average almost twice as wide as those among the commodity series.

<sup>1</sup> *Bulletin No. 173*, p. 98. The corresponding figure for the Bureau's new weighted index is 26 per cent (*Bulletin No. 181*, p. 16).

TABLE IV

AVERAGE DIFFERENCES AMONG TEN INDEX NUMBERS OF STOCKS AND AMONG FOUR INDEX NUMBERS OF COMMODITIES: PERCENTAGES OF RISE OR FALL FROM ONE YEAR TO THE NEXT

(Based upon Table II above, and upon *Bulletin of the U.S. Bureau of Labor Statistics, No. 173*, p. 108; and *No. 181*, p. 19)  
(The figures in each space of the table show the average number of points in the scale of relative prices on the preceding-year base between the yearly entries of the two index numbers which are indicated by the descriptive title above and by the numeral in the left-hand margin)

## I. TEN STOCK INDEXES, 1890-1915

FORTY TRANSPORTATION STOCKS											
NUMERALS DESIGNATING THE SEVERAL INDEX NUMBERS	Series Made from the Average Actual Prices Shown in Table I			Simple Averages			Weighted Averages			TENTY RAILWAY STOCKS; <i>Wall Street Journal</i> SERIES FROM TABLE I	ALL STOCKS SOLD ON THE NEW YORK STOCK EXCHANGE FROM TABLE I
	Series Made from the Average Actual Prices Shown in Table I			Simple Averages			Weighted Averages				
	Series Made from the Average Actual Prices Shown in Table I			Simple Averages			Weighted Averages				
	Series Made from the Average Actual Prices Shown in Table I			Simple Averages			Weighted Averages				
	Medians (1)	Arithmetic Means (2)	Geometric Means (3)	Medians (4)	Arithmetic Means (5)	Stock Outstanding (6)	Earnings (7)	Number of Shares Sold (8)	(9)	(10)	
(1).....	.....	4.3	3.1	8.5	2.6	3.1	3.3	5.3	3.8	9.7	
(2).....	4.3	.....	2.3	7.1	5.7	6.6	6.6	5.7	6.3	12.9	
(3).....	3.1	2.3	.....	7.8	4.8	5.7	5.6	5.3	6.0	11.9	
(4).....	8.5	7.1	7.8	.....	9.6	10.2	10.4	8.1	9.8	16.2	
(5).....	2.6	5.7	4.8	9.6	.....	1.3	1.2	4.9	3.1	8.1	
(6).....	3.1	6.6	5.7	10.2	1.3	.....	0.6	5.1	2.7	7.3	
(7).....	3.3	6.6	5.6	10.4	1.2	0.6	.....	5.2	4.4	7.4	
(8).....	5.3	5.7	5.3	8.1	4.9	5.1	5.2	5.2	4.6	10.6	
(9).....	3.8	6.3	6.0	9.8	3.1	2.7	4.4	4.4	4.6	7.7	
(10).....	9.7	12.9	11.9	16.2	8.1	7.3	7.4	10.6	7.7	.....	
Averages.....	4.9	6.4	5.8	9.7	4.6	4.7	5.0	6.1	5.4	10.2	
Averages excluding Nos. 4 and 10.....	3.6	5.4	4.7	.....	3.4	3.6	3.8	5.2	4.4	.....	

TABLE IV—*Continued*

## 2. FOUR COMMODITY INDEXES, 1890-1914

NUMERALS DESIGNATING THE SEVERAL INDEX NUMBERS	BUREAU OF LABOR STATISTICS		BRADSTREET'S INDEX NUMBER REDUCED TO PERCENTAGES OF YEARLY RISE OR FALL	DUN'S INDEX NUMBER REDUCED TO PERCENTAGES OF YEARLY RISE OR FALL
	Percentages of Yearly Rise or Fall in the Weighted Aggregates of Actual Prices (1)	Series Made from Arithmetic Means of Percentages of Yearly Rise or Fall (2)		
(1).....	.....	1.9	2.4	2.0
(2).....	1.9	.....	1.9	2.9
(3).....	2.4	1.9	.....	2.8
(4).....	2.0	2.9	2.8	.....
Averages.....	2.1	2.2	2.4	2.6

On passing to the 1890-99 base, we get more commodity series for comparison, among them five pairs (Nos. 2 and 3, 1 and 5, 2 and 4, 3 and 4, 3 and 5) that agree a little more closely than the most similar pair of stock indexes (Nos. 6 and 7, average difference 2.1 points). Still there remain five pairs of the latter (Nos. 6 and 7, 5 and 7, 5 and 6, 5 and 10, 7 and 10) that agree as well or better with each other than do some pairs of the commodity indexes. The important thing to notice here, however, is that the 1890-99 base gives much more divergent results than the preceding-year base. Especially noteworthy is the fact that the fixed-base series tend to scatter more and more with the lapse of time, whereas the moving-base series show no such tendency. Table VI establishes this result. Percentages of change from prices in the year before do indeed scatter more in years of rising than in years of falling prices, but these divergences are not cumulative.

Since the stock indexes run upon a higher level than the commodity indexes in most years since 1890, they may be expected to show wider divergences than the commodity indexes. A supplementary computation is necessary to prove that these divergences are greater not only absolutely but also relatively. To this end it suffices to select from each of our sets of index numbers some single series, and then to compute each year the percentages by which it differs from each of its fellow-series. Table VII presents the results of this experiment. The series chosen as the standards

TABLE V

AVERAGE DIFFERENCES AMONG ELEVEN INDEX NUMBERS OF STOCKS AND AMONG EIGHT INDEX NUMBERS OF COMMODITIES: RELATIVE PRICES ON THE 1890-99 BASE

(Based upon Table III above and upon *Bulletin of the U.S. Bureau of Labor Statistics, No. 173*, pp. 96, 105, and *No. 181*, p. 16)

(The figures in each space of the table show the average number of points in the scale of relative prices on the 1890-99 base between the yearly entries of the two index numbers which are indicated by the descriptive title above and by the numeral on the left-hand margin)

## I. ELEVEN STOCK INDEXES, 1890-1915

NUMERALS DESIGNATING THE SEVERAL INDEX NUMBERS	FORTY COMMON STOCKS											FIFTY STOCKS: 40 COMMON AND 10 PREFERRED; <i>Wall Street Journal</i> ARITHMETIC MEANS OF RELATIVE PRICES	TWENTY RAILWAY STOCKS; <i>Wall Street Journal</i> ARITHMETIC MEANS OF RELATIVE PRICES	ALL STOCKS SOLD ON THE NEW YORK STOCK EXCHANGE	
	Series Made from the Relative Prices of Each Stock			Series Made from the Average Actual Prices Shown in Table I					Weighted Averages						
	Simple Averages			Arithmetic Means		Stock Outstanding (6)	Earnings (7)	Number of Shares Sold (8)	Arithmetic Means Weighted by						
	Geometric Means (3)	Medians (4)	Arithmetic Means (5)												
	Medians (1)	Arithmetic Means (2)													
(1).....	.....	26	11	52	14	19	18	44	17	17	39				
(2).....	26	.....	16	27	28	35	34	21	10	20	59				
(3).....	11	16	.....	43	16	22	20	35	8	18	45				
(4).....	52	27	43	.....	54	61	60	17	36	54	85				
(5).....	14	28	16	54	.....	7	6	46	19	6	31				
(6).....	19	35	22	61	7	.....	2	53	26	8	24				
(7).....	18	34	20	60	6	2	.....	52	24	7	26				
(8).....	44	21	35	17	46	53	52	.....	29	47	77				
(9).....	17	10	8	36	19	20	24	29	.....	19	49				
(10).....	17	29	18	54	6	8	7	47	19	.....	31				
(11).....	39	59	45	85	31	24	26	77	49	31	.....				
Averages.....	26	29	23	49	23	26	25	42	24	24	47				
Averages excluding Nos. 4 and 11..	21	25	18	.....	18	22	20	41	19	19	.....				



TABLE V—*Continued*  
2. EIGHT COMMODITY INDEXES, 1890-1913

NUMERALS DESIGNATING THE SEVERAL INDEX NUMBERS	BUREAU OF LABOR STATISTICS SERIES			BRADSTREET'S INDEX NUMBER SHIFTED TO THE 1890-99 BASE; 96 COMMODITIES	GIBSON'S INDEX NUMBER, ORIGINAL LIST; 50 COMMODITIES	DUN'S INDEX NUMBER SHIFTED TO THE 1890-99 BASE; 310 (?) COMMODITIES	GIBSON'S PRESENT INDEX NUMBER SHIFTED TO THE 1890-99 BASE; 22 COMMODITIES	Analyst INDEX NUMBER WITH MISSING ENTRIES SUPPLIED; 25 COMMODITIES
	Relatives Made from Weighted Aggregates of Actual Prices (1)	Arithmetic Means of Relative Prices of 242 = Commodities (2)	Arithmetic Means of Relative Prices of 145 Commodities (3)					
(1).....	.....	2.8	2.7	3.9	1.5	3.9	5.5	5.3
(2).....	2.8	.....	1.0	1.9	2.5	5.5	7.2	6.6
(3).....	2.7	1.0	.....	2.0	2.0	5.3	6.8	6.3
(4).....	3.9	1.9	2.0	.....	3.5	6.6	7.0	6.7
(5).....	1.5	2.5	2.0	3.5	.....	4.1	5.9	5.5
(6).....	3.9	5.5	5.3	6.6	4.1	.....	4.5	6.1
(7).....	5.5	7.2	6.8	7.0	5.9	.....	.....	3.9
(8).....	5.3	6.6	6.3	6.7	5.5	4.5	3.9	.....
Averages.....	3.7	3.9	3.7	4.5	3.6	5.1	5.8	5.8

for stock indexes are the simple arithmetic means of the actual prices of forty stocks and the two sets of relatives made from them (No. 3 in Table I, No. 5 in Tables II and III). For commodities the Bureau's new weighted average is used. This form of comparison does indeed diminish the differences among the stock indexes more than it diminishes the differences among the commodity indexes, and therefore it diminishes the differences between the two sets of differences. It gives us again cases in which two index numbers of stock prices agree with each other better than do certain index numbers of commodity indexes. Nevertheless, it still leaves the stock series considered as one group much less harmonious than the commodity series considered as a second group.

TABLE VI

AVERAGE DIFFERENCES AMONG VARIOUS INDEX NUMBERS OF STOCKS, BY PERIODS OF YEARS

(Based upon the data used in making Tables IV and V)

	SERIES COMPUTED UPON THE PRECEDING-YEAR BASE		SERIES COMPUTED UPON THE 1890-99 BASE	
	All Series in Table II	All Series except Nos. 4 and 10	All Series in Table III	All Series except Nos. 4 and 11
1890-1899.....	7.8	4.3	8.6	5.2
1900-1909.....	5.7	4.5	41.1	28.6
1910-1915.....	4.3	3.5	49.1	40.6
1890-1915.....	6.3	4.3	30.4	22.4

So far our comparisons of index numbers of the prices of stocks and of commodities indicate (1) that the stock indexes agree rather better than the commodity indexes concerning the direction in which prices move from one year to the next; (2) that they show these fluctuations to be of much greater amplitude in the case of stocks than in the case of commodities; (3) that in measuring the amplitude of these fluctuations over long periods of time they differ from one another much more than do commodity indexes, and (4) that when the entries for each successive year are compared the stock indexes continue to show wider discrepancies, not only absolutely but also relatively, than commodity indexes show.

TABLE VII

AVERAGE YEARLY DIFFERENCES BETWEEN CERTAIN INDEX NUMBERS OF STOCKS AND OF COMMODITIES IN PERCENTAGES OF THE SEVERAL SERIES SELECTED AS STANDARDS

Series with Which Comparisons Are Made	STOCKS		COMMODITIES			
	Actual Prices, Table I; Differences Between Series 3 and the Series Named	Relative Prices on Preceding-Year Base, Table II; Differences between Series 5 and the Series Named	Relative Prices on 1800-99 Base, Table III; Differences between Series 5 and the Series Named	Series with Which Comparisons Are Made	Relative Prices on Preceding-Year Base; Differences between B.L.S. Weighted Series and the Series Named	Relative Prices on 1800-99 Base; Differences between the B.L.S. Weighted Series and the Series Named
	Per Cent	Per Cent	Per Cent		Per Cent	Per Cent
Medians of relative prices	.....	2.5	8.6	B.L.S. simple average. . . .	1.9	2.3
Arithmetic means of relative prices. . . . .	.....	5.1	16.8	B.L.S. revised, 145 commodities. . . . .	.....	2.4
Geometric means of actual prices. . . . .	.....	4.6	9.9	Bradstreet's index number	2.4	3.4
Medians of actual prices	33.8	9.5	33.6	Gibson's index—original list. . . . .	.....	1.5
Arithmetic means weighted by stock outstanding. . .	27.2	1.3	4.0	Dun's index number. . . .	1.9	3.3
Arithmetic means weighted by earnings. . . . .	11.1	1.0	3.2	Gibson's present index number. . . . .	.....	4.9
Arithmetic means weighted by number of shares sold	23.2	4.7	27.7	<i>Annalist</i> index number. . .	.....	5.0
Forty common and ten preferred stocks. . . . .	16.8	.....	11.2			
<i>Wall Street Journal</i> series of twenty railway stocks	.....	3.1	3.9			
All stocks sold on the New York Exchange. . . . .	12.9	8.1	18.6			
	16.2					

How disconcerting should these results be rated? Do they mean that the attempt to measure fluctuations in the prices of stocks had better be abandoned as fruitless? Or that the present stock indexes at least are untrustworthy? Or that some of the series are sound and the others misleading? If there is a choice, which series is most accurate?

To answer these questions, we must trace the discrepancies among these series back to their sources in different methods of construction and in the peculiarities of the data to which these methods are applied.

#### V. TECHNICAL DIFFERENCES OF CONSTRUCTION AND THEIR EFFECTS UPON THE RESULTS

Aside from the purely formal differences between averages of actual prices and these same averages reduced to relatives,<sup>1</sup> the stock index numbers before us differ technically in respect to: (1) the stocks included, (2) the kind of average struck, (3) weighting, (4) the use of averages of relative prices or relatives of average prices, and (5) the base on which relative prices are computed.<sup>2</sup> In certain cases two or three series are alike in all save one of these five respects; in most cases they differ in two or more respects.

To trace out the differences in results produced by all of these differences of method in all of their combinations would be tedious.

<sup>1</sup> That is, the differences between the series in Table I and the corresponding series in Table II, columns 3-10, and Table III, columns 3-8, 10 and 11.

<sup>2</sup> More in detail: (1) Four different samples of stocks are used: (a) common stocks in forty transportation companies, (b) these forty stocks with ten preferred railway stocks added, (c) the *Wall Street Journal's* changing list of twenty railway stocks, common and preferred, (d) the *Financial Review's* figures for all stocks sold each year on the New York Exchange. (2) Three kinds of average are employed, arithmetic means, geometric means, and medians. (3) Both haphazard and systematic weights are tried. In the "simple" averages the relative influence exercised upon the results by each stock is left to chance, and cannot be determined without a special inquiry (see Section X, below). Four sets of systematic weights are employed—three sets that remain constant throughout the whole period and one set that changes every year (in the series from the *Financial Review*). (4) In some series the actual prices of each stock are turned into relative prices at the outset and then these relatives are averaged. In other series the actual prices are first averaged and then the averages are turned into relatives. (5) In both of the procedures just described, the relatives are worked out on two different bases—one changing every year, the other fixed.

But the self-explanatory schedules which follow provide data for a few significant conclusions.

Series in Table II Which Differ <i>Only</i> in Respect to:		Average Differences in Results
1. Sampling		
Nos. 5 and 9—40 stocks vs. 20 stocks . . . . .		3.1 points
2. Forms of average used		
Averages of relative prices		
Nos. 1 and 2—Medians vs. arithmetic means . . . . .		4.3
Nos. 1 and 3—Medians vs. geometric means <sup>1</sup> . . . . .		3.1
Nos. 2 and 3—Arithmetic vs. geometric means . . . . .		2.3
Relatives of average actual prices		
Nos. 3 and 4—Geometric means vs. medians . . . . .		7.8
Nos. 3 and 5—Geometric vs. arithmetic means . . . . .		4.8
Nos. 4 and 5—Medians vs. arithmetic means . . . . .		9.6
3. Weighting		
Nos. 6 and 7—Stocks outstanding vs. earnings as weights . . .		0.6
Nos. 6 and 8—Stock outstanding vs. shares sold as weights . .		5.1
Nos. 7 and 8—Earnings vs. shares sold as weights . . . . .		5.2
4. Use of averages of relative prices vs. relatives of average actual prices:		
Nos. 1 and 4—Medians . . . . .		8.5
Nos. 2 and 5—Arithmetic means . . . . .		5.7

Series in Table III Which Differ <i>Only</i> in Respect to:		Average Differences in Results
1. Sampling		
Nos. 5 and 10—40 stocks vs. 20 stocks . . . . .		6 points
Nos. 2 and 9—40 stocks vs. 50 stocks . . . . .		10
2. Forms of average used		
Averages of relative prices		
Nos. 1 and 2—Medians vs. arithmetic means . . . . .		26
Nos. 1 and 3—Medians vs. geometric means . . . . .		11
Nos. 2 and 3—Arithmetic vs. geometric means . . . . .		16
Relatives of average actual prices		
Nos. 3 and 4—Geometric means vs. medians . . . . .		43
Nos. 3 and 5—Geometric vs. arithmetic means . . . . .		16
Nos. 4 and 5—Medians vs. arithmetic means . . . . .		54

<sup>1</sup> The geometric means in Tables II and III were actually computed from geometric means of actual prices. But when this form of average is used, it makes no difference whether relative prices are made first and then averaged, or whether actual prices are first averaged and then reduced to relatives. Hence in these two schedules, the geometric means are fairly included under both captions, "Averages of relative prices" and "Relatives of average actual prices."

3. Weighting	
Nos. 6 and 7—Stock outstanding vs. earnings as weights . . . . .	2
Nos. 6 and 8—Stock outstanding vs. shares sold as weights . . .	53
Nos. 7 and 8—Earnings vs. shares sold as weights . . . . .	52
4. Use of averages of relative prices vs. relatives of average prices	
Nos. 1 and 4—Medians . . . . .	52
Nos. 2 and 5—Arithmetic means . . . . .	28

Among these differences in method it will be seen that differences of sampling produce on the whole the slightest differences in results. The form of average used counts for more, especially when the averages are struck from the actual prices. Whether weighting makes very little or very great differences depends on whether the weights themselves differ slightly or widely. Stock outstanding and earnings give weights that are much the same, while the number of shares sold yields a radically different set (see Table XXI, below). When the same sample, the same form of average, and no systematic weights are used, it also makes a great difference in the results whether relatives are made first and then averaged or averages are made first and then turned into relatives.

As for the fifth difference noted at the beginning of this section, that in the base on which relatives are computed, it affects only the series that are made by averaging relative prices—that is, Nos. 1 and 2 in Table II, and Nos. 1, 2, and 9 in Table III. To determine its effect upon the results requires a new computation. Percentages of annual rise or fall in price may be made from the 1890–99 relatives of Table III and set beside the year-to-year relatives of Table II. When this is done the new figures based on column 1 of Table III (medians) show average differences of 4.7 points from the corresponding series in Table II. In the case of the arithmetic means (column 2 in both tables) the average differences in results are less—2.4 points. It may be pointed out further that this difference in the base used often alters the relationship in results among the several series. To give but one example: On the preceding-year base the medians of relative prices differ least in results from the relatives made from arithmetic means of actual prices (Table II, Nos. 1 and 5); whereas on the 1890–99 base these medians differ least in results from the geometric means (Table III, Nos. 1 and 3).

One wider generalization may be stated. In a majority of cases the series that resemble each other most closely in point of construction are not the series that resemble each other most closely in point of results.<sup>1</sup>

In these relations between methods and results, we have another difference between index numbers of stocks and commodities. Much the most important factor in producing differences among the results of commodity index numbers is sampling. That is, differences in the commodities included count for much more than any other difference in construction.<sup>2</sup> If we may judge from the present material, however, when one deals with stocks, differences in sampling count for less than differences in averaging, in weighting, or in the stage at which actual prices are turned into relatives. And more than half of the indexes find their closest analogues in series to which they are not closely related in construction.

These conclusions seem puzzling. The relatively moderate difference produced by using twenty common and preferred stocks in place of forty common stocks, or by adding ten preferred stocks to forty common, suggests that the three available samples must

<sup>1</sup> Again the facts may be presented succinctly in schedule form:

TABLE II SERIES	DIFFERS LEAST IN		TABLE III SERIES	DIFFERS LEAST IN	
	Construc- tion from	Results from		Construc- tion from	Results from
No. 1.....	Nos. 2, 3	No. 5	No. 1.....	Nos. 2, 3	No. 3*
" 2.....	" 1, 3	" 3*	" 2.....	" 1, 3, 9	" 9*
" 3.....	" 1, 2	" 2*	" 3.....	" 1, 2, 4, 5	" 9
" 4.....	" 3, 5	" 2	" 4.....	" 3, 5	" 8
" 5.....	" 3, 4, 9	" 7	" 5.....	" 3, 4	Nos. 7, 10
" 6.....	" 7, 8	" 7*	" 6.....	" 7, 8	No. 7*
" 7.....	" 6, 8	" 6*	" 7.....	" 6, 8	" 6*
" 8.....	" 6, 7	" 9	" 8.....	" 6, 7	" 4
" 9.....	No. 5	" 6	" 9.....	No. 2	" 3
" 10.....	" 8	" 6	" 10.....	" 5	" 5*
			" 11.....	" 8	" 6

Nine of the series (marked by asterisks) agree best in results with some one of the series to which they are most closely related in construction; but twelve series agree better with some stranger, than with their next of kin. And of the first nine cases, four hardly count, because the differences in the weights based upon stock outstanding and upon earnings (Nos. 6 and 7 in both tables) are so slight as not to constitute an important difference in method.

<sup>2</sup> See the comparison of seven commodity index numbers for the United States, 1890-1913, in *Bulletin of the Bureau of Labor*, No. 173, pp. 93-106, especially pp. 103-5.

be rather similar.<sup>1</sup> That would be reassuring. But the wider differences produced by working up the same sample in various ways suggest that a given sample must contain within itself most dissimilar elements. And that would be disconcerting. Is the explanation perhaps that the different samples are alike only in that none of them are composed of homogeneous data? If that is the case, can we make any series from any sample that possesses high representative value? These questions force us to push our inquiry home by scrutinizing the original data from which our stock indexes are made.

#### VI. CHARACTERISTICS AND DIFFICULTIES OF THE DATA FROM WHICH STOCK INDEXES ARE MADE

To present in full the original data used in making even one set of the stock index numbers presented above—the set including forty common stocks—would fill many pages and give no clear impression. For these data consist of some 24,000 quotations from the Stock Exchange lists, showing monthly for twenty-six years the highest and lowest prices of each stock.<sup>2</sup> To reduce the bulk of material and still to keep its essential character, the twenty-four quotations for each stock each year have been made into a single arithmetic mean. But even that step leaves 1,040 prices to deal with—a bewildering number when one tries to see the whole array at once. To bring the data within manageable compass there is needed a series of systematic presentations each of which comprises all the material, while throwing some significant feature into relief.

Tables VIII, IX, and X treat the individual stocks, showing respectively their actual prices, percentages of change in price from one year to the next, and relative prices on the 1890-99 base.

<sup>1</sup> The fourth sample—all stocks sold on the New York Stock Exchange—is not included in the analysis, because the series in which it is used (No. 10 in Table II, No. 11 in Table III) differs from all the others not only in sampling, but also in weighting, and we have no means of separating the effects of these two differentiating characteristics.

<sup>2</sup> If no quotations were missing, there would be 24,960 prices. But in 1914 the Stock Exchange was closed from August to November, and in every other year one or more stocks failed to be quoted in one or more months. In recent years, however, these gaps have been filled by bid and asked prices, or by interpolations.



TABLE VIII  
AVERAGE AND EXTREME PRICES OF FORTY COMMON STOCKS IN 1890-1915. ARITHMETIC MEANS

	AVERAGE PRICES IN				EXTREME PRICES IN 1890-1915		YEARS IN WHICH EXTREME YEARLY AVERAGE PRICES OCCURRED	
	1890-1915 (1)	1890-1899 (2)	1900-1909 (3)	1910-1915 (4)	Highest Yearly Average (5)	Lowest Yearly Average (6)	Highest (7)	Lowest (8)
Forty stocks—average.....	\$ 80.69	\$ 54.80	\$ 97.92	\$ 95.12	\$120.10	\$ 46.36	1909	1896
North Atlantic railways—average.....	94.17	91.96	105.65	78.72	122.58	60.38	1902	1914
New York, New Haven & Hartford.....	178.82	207.92	191.16	109.77	256.20	60.80	1890	1914
New York, Ontario & Western.....	28.71	17.54	37.01	33.48	52.50	14.20	1905	1896
New York Central.....	116.24	107.42	133.32	102.55	159.30	85.30	1902	1914
Pennsylvania.....	121.24	109.37	134.85	118.40	155.10	100.20	1902	1894
Erie.....	25.82	17.56	31.93	29.40	45.90	10.80	1905	1895
Anthracite coal railways—average.....	180.51	104.00	200.85	274.19	313.13	92.20	1912	1898
Central Railroad of New Jersey.....	181.13	108.33	179.06	305.98	367.80	89.10	1912	1897
Delaware & Hudson.....	151.52	126.68	171.89	159.02	215.90	107.00	1906	1898
Delaware, Lackawanna & Western.....	310.63	154.45	367.83	475.73	589.60	137.50	1909	1891
Reading.....	78.76	26.54	84.60	156.03	166.20	14.80	1912	1895
Middle-western railways—average.....	48.13	35.60	59.68	49.75	74.71	29.05	1902	1896
Cleveland, Cincinnati, Chicago & St. Louis.....	61.00	47.97	80.68	49.95	102.00	29.60	1902	1914
Wheeling & Lake Erie.....	13.26	16.50	15.14	4.75	35.10	1.90	1890	1897
New York, Chicago & St. Louis.....	33.14	14.80	41.95	50.53	66.10	12.60	1906	1896
Wabash.....	12.26	8.77	18.67	7.40	28.40	.60	1902	1915
Pittsburgh, Cincinnati, Chicago & St. Louis.....	59.29	24.89	75.59	89.60	106.90	13.10	1912	1890
Canada Southern.....	60.42	52.69	67.24	61.92	87.60	47.70	1902	1896
Lake Erie & Western.....	23.14	18.35	34.72	11.83	63.10	6.00	1902	1914
Illinois Central.....	122.50	100.83	144.45	122.05	174.30	91.80	1906	1894
Northwestern railways—average.....	65.28	41.43	88.42	66.48	109.20	32.81	1902	1894
Chicago, Milwaukee & St. Paul.....	114.05	79.20	152.62	107.85	178.20	60.50	1905	1894



TABLE IX  
CHANGES IN THE PRICES OF FORTY COMMON STOCKS FROM ONE YEAR TO THE NEXT: 1890-91 TO 1914-15  
ARITHMETIC MEANS

	NUMBER OF YEARS IN WHICH AVERAGE ANNUAL PRICES			AVERAGE PERCENTAGE OF CHANGE IN PRICES FROM ONE YEAR TO THE NEXT				EXTREME PERCENTAGES OF CHANGE	
	Rose (1)	Fell (2)	Did Not Change (3)	All Cases (4)	Cases of Rise (5)	Cases of Fall (6)	Greatest Rise (7)	Greatest Fall (8)	
Forty stocks—average.....	12	13	.....	Per Cent 20.1	Per Cent 26.9	Per Cent 14.8	Per Cent 61.7	Per Cent 23.8	
North Atlantic railways—average.....	11	14	.....	13.8	17.4	11.0	53.8	24.5	
New York, New Haven & Hartford.....	10	15	.....	9.8	6.6	11.9	17.6	40.3	
New York, Ontario & Western.....	11	14	.....	18.2	25.7	12.3	78.6	26.6	
New York Central.....	11	14	.....	10.0	11.6	8.8	26.4	20.3	
Pennsylvania.....	12	12	I	6.1	6.8	5.9	14.7	16.4	
Erie.....	11	14	.....	25.1	30.3	16.3	193.0	45.7	
Anthracite coal railways—average.....	13	12	.....	16.2	21.4	10.4	54.1	21.3	
Central Railroad of New Jersey.....	14	11	.....	16.8	20.9	11.6	108.2	36.4	
Delaware & Hudson.....	11	14	.....	8.7	10.5	7.2	42.6	21.3	
Delaware, Lackawanna & Western.....	13	12	.....	11.2	16.1	6.0	44.3	20.4	
Reading.....	13	12	.....	28.0	38.2	16.9	121.3	57.3	
Middle-western railways—average.....	11	14	.....	22.0	29.7	16.9	78.4	31.0	
Cleveland, Cincinnati, Chicago & St. Louis.....	12	13	.....	18.8	19.4	18.2	54.0	36.7	
Wheeling & Lake Erie.....	9	16	.....	39.9	61.0	28.1	285.7	78.4	
New York, Chicago & St. Louis.....	12	11	2	23.2	33.4	16.3	170.3	44.0	
Wabash.....	9	16	.....	28.2	34.5	24.6	157.7	59.7	
Pittsburgh, Cincinnati, Chicago & St. Louis.....	14	11	.....	23.2	30.4	14.1	129.4	30.2	
Canada Southern.....	11	14	.....	7.7	9.7	6.2	36.5	23.3	
Lake Erie & Western.....	9	15	I	27.2	41.8	20.2	106.9	45.0	
Illinois Central.....	13	12	.....	7.6	7.7	7.6	23.5	19.6	



TABLE X

AVERAGE AND EXTREME RELATIVE PRICES OF FORTY COMMON STOCKS IN 1890-1915. AVERAGE ACTUAL PRICES IN 1890-99 = 100  
ARITHMETIC MEANS

	AVERAGE RELATIVE PRICES IN				EXTREME RELATIVE PRICES IN 1890-1915		YEARS IN WHICH EXTREME YEARLY AVERAGE RELATIVE PRICES OCCURRED	
	1890-1915 (1)	1890-1899 (2)	1900-1909 (3)	1910-1915 (4)	Highest Yearly Average (5)	Lowest Yearly Average (6)	Highest (7)	Lowest (8)
Forty stocks—average.....	173	100	219	218	277	77	1909	1896
North Atlantic railways—average.....	123	100	146	123	186	86	1905	1896
New York, New Haven & Hartford.....	86	100	92	53	123	29	1890	1914
New York, Ontario & Western.....	164	100	211	191	300	81	1905	1896
New York Central.....	108	100	124	95	148	79	1902	1914
Pennsylvania.....	111	100	123	108	142	92	1902	1894
Erie.....	147	100	181	167	261	61	1905	1895
Anthracite coal railways—average.....	196	100	215	326	364	84	1912	1898
Central Railroad of New Jersey.....	167	100	165	282	340	82	1912	1897
Delaware & Hudson.....	120	100	136	126	170	84	1906	1898
Delaware, Lackawanna & Western.....	201	100	238	308	382	89	1909	1891
Reading.....	297	100	319	389	627	56	1912	1895
Middle-western railways—average.....	146	100	189	153	256	76	1902	1897
Cleveland, Cincinnati, Chicago & St. Louis.....	127	100	168	104	213	62	1902	1914
Wheeling & Lake Erie.....	80	100	92	29	213	12	1890	1897
New York, Chicago & St. Louis.....	224	100	277	341	447	85	1906	1896
Wabash.....	139	100	212	84	323	7	1902	1915
Pittsburgh, Cincinnati, Chicago & St. Louis.....	238	100	303	360	429	53	1912	1890
Canada Southern.....	115	100	128	117	166	91	1902	1896
Lake Erie & Western.....	126	100	190	65	345	33	1902	1914
Illinois Central.....	122	100	143	121	173	91	1906	1894

Northwestern railways—average.....	179	100	248	195	332	76	1902	1804
Chicago, Milwaukee & St. Paul.....	144	100	193	136	225	76	1905	1804
Chicago & Northwestern.....	129	100	163	119	204	86	1902	1805
Chicago, St. Paul, Minneapolis & Omaha.....	208	100	280	255	369	56	1905	1801
Duluth, South Shore & Atlantic.....	154	100	216	141	320	49	1906	1808
Iowa Central.....	182	100	296	130	493	70	1902	1914
Minneapolis & St. Louis.....	195	100	341	109	564	30	1902	1801
Wisconsin Central.....	241	100	241	474	669	20	1911	1898
Southern railways—average.....	215	100	276	307	358	67	1909	1896
Chesapeake & Ohio.....	198	100	223	319	386	75	1910	1896
Norfolk & Western.....	458	100	569	870	937	30	1912	1895
Louisville & Nashville.....	163	100	190	221	249	75	1912	1896
Missouri Pacific.....	136	100	204	82	258	20	1902	1915
Missouri, Kansas & Texas.....	163	100	214	183	330	59	1909	1890
Texas & Pacific.....	174	100	253	165	350	63	1902	1893
Pacific railways—average.....	235	100	293	363	445	64	1909	1896
Atchison, Topeka & Santa Fe.....	290	100	369	473	524	40	1909	1894
Denver & Rio Grande.....	156	100	226	134	318	54	1909	1915
Southern Pacific.....	233	100	268	394	482	66	1909	1897
Union Pacific.....	320	100	412	533	659	26	1909	1896
Canadian Pacific.....	176	100	189	279	345	70	1912	1895
Express companies, etc.—average.....	134	100	154	113	178	85	1906	1896
American Express.....	139	100	174	144	218	82	1910	1915
United States Express.....	148	100	194	152	241	80	1902	1896
Wells Fargo.....	136	100	192	101	314	68	1909	1914
Pacific Mail Steamship.....	104	100	116	89	154	56	1899	1894
Western Union.....	94	100	95	81	107	65	1905	1908

TABLE XI  
DECILES OF THE ACTUAL PRICES OF FORTY COMMON STOCKS. BY YEARS: 1890-1915

Years	Lowest Price	First Decil	Second Decil	Third Decil	Fourth Decil	Median	Sixth Decil	Seventh Decil	Eighth Decil	Ninth Decil	Highest Price
1890.....	\$ 6.00	\$10.25	\$17.25	\$21.00	\$31.20	\$40.20	\$68.35	\$ 79.55	\$ 107.95	\$120.00	\$256.20
1891.....	5.80	12.80	16.00	19.10	31.15	35.75	62.65	78.65	105.15	124.10	234.90
1892.....	10.00	12.15	17.00	23.70	31.90	45.50	59.45	84.25	111.55	133.95	240.90
1893.....	7.40	8.70	12.85	16.20	19.25	26.65	46.55	72.75	103.25	119.30	215.10
1894.....	5.30	7.75	9.90	14.60	16.10	19.60	43.75	63.15	99.60	115.40	185.70
1895.....	3.60	8.35	11.70	14.85	18.55	24.30	42.45	61.30	99.45	110.10	176.80
1896.....	2.60	7.10	10.25	14.20	15.30	20.15	40.50	65.30	94.90	108.05	192.80
1897.....	1.90	7.45	12.80	14.15	18.35	23.70	46.20	75.10	100.50	112.45	175.30
1898.....	2.00	8.30	13.20	15.20	21.45	32.05	48.40	87.55	106.80	124.15	189.80
1899.....	4.90	12.30	16.75	21.80	31.80	49.25	62.60	99.60	123.15	139.75	212.50
1900.....	5.10	11.95	17.45	25.35	35.55	52.15	60.85	103.25	123.00	143.85	212.10
1901.....	9.20	23.20	37.00	42.30	50.50	76.80	97.20	120.55	154.35	178.70	222.60
1902.....	16.60	27.90	42.85	49.60	65.25	91.70	108.75	139.55	166.30	221.00	273.30
1903.....	11.90	23.95	20.00	33.30	58.35	74.30	93.00	136.80	143.80	188.10	248.80
1904.....	8.70	20.25	28.20	31.45	55.70	71.50	94.30	122.75	148.90	185.65	283.60
1905.....	15.80	27.00	35.35	51.75	68.00	90.10	107.40	144.05	173.25	212.35	409.30
1906.....	18.30	27.75	37.60	53.70	74.30	93.20	129.60	155.30	177.95	219.65	504.40
1907.....	10.40	17.35	27.05	36.55	64.10	74.60	99.85	123.00	144.75	174.75	455.70
1908.....	8.20	17.40	30.10	53.65	72.60	92.25	128.95	159.10	172.55	211.55	589.60
1909.....	9.60	27.20	39.10	53.65	72.60	92.25	128.95	159.10	172.55	211.55	589.60
1910.....	5.30	19.40	30.95	47.60	67.70	98.20	117.80	139.80	153.35	181.45	551.60
1911.....	4.20	16.15	29.25	44.00	63.25	95.30	111.35	137.25	149.70	201.90	537.00
1912.....	5.80	13.25	25.90	38.60	62.25	96.45	111.95	132.55	148.25	186.05	549.10
1913.....	3.50	8.20	20.85	30.95	52.65	78.35	100.90	109.85	131.75	159.25	404.00
1914.....	1.20	6.20	14.05	23.95	43.10	65.35	87.05	100.20	127.65	148.70	389.50
1915.....	.60	8.00	11.20	28.80	41.55	67.90	91.15	104.20	120.20	149.40	423.20
Averages											
1890-1899.....	4.95	9.52	13.77	17.48	23.51	31.72	52.09	76.72	105.23	121.63	207.92
1900-1909.....	11.38	22.31	31.79	41.20	60.45	78.90	101.77	131.23	154.91	191.22	371.13
1910-1915.....	3.43	11.87	22.03	35.65	55.08	83.59	103.37	120.64	138.48	171.13	475.73
1890-1915.....	7.07	14.97	22.61	30.79	45.00	61.84	83.03	107.80	132.00	159.80	332.50

TABLE XII  
DECILS OF THE PERCENTAGES OF RISE (+) OR FALL (—) IN THE PRICES OF FORTY COMMON STOCKS FROM ONE YEAR TO THE NEXT.  
BY YEARS: 1890-91 TO 1914-15

Years	Greatest Fall or Least Rise	First Decil	Second Decil	Third Decil	Fourth Decil	Median	Sixth Decil	Seventh Decil	Eighth Decil	Ninth Decil	Greatest Rise or Least Fall
1890-91.....	-27.80	-19.45	-10.20	-7.90	-6.25	-3.30	-2.15	-1.40	-	+ 3.80	+ 96.10
1891-92.....	-25.70	-11.50	-5.05	+ 2.55	+ 3.95	+ 6.90	+ 9.85	+14.55	+ 21.90	+56.60	+136.20
1892-93.....	-57.30	-42.25	-35.85	-30.80	-24.00	-16.75	-14.20	-11.60	-7.85	-5.30	+ 3.70
1893-94.....	-64.90	-31.20	-21.50	-10.25	-13.60	-10.85	-8.00	-3.90	-	-	+16.70
1894-95.....	-43.70	-19.70	-8.00	-1.60	+ 2.30	+ 4.25	+ 5.60	+10.40	+13.05	+31.35	+142.70
1895-96.....	-46.90	-26.70	-18.65	-15.70	-13.05	-10.45	-5.95	-2.70	+ 1.85	+ 9.85	+58.30
1896-97.....	-78.40	-11.80	-5.00	+ 0.75	+ 2.60	+ 6.15	+ 9.70	+13.35	+24.25	+44.25	+128.10
1897-98.....	-19.40	+ 7.10	+ 2.00	+ 3.00	+ 6.50	+ 9.65	+13.00	+14.40	+20.45	+31.70	+120.40
1898-99.....	-0.30	+ 7.00	+10.35	+13.00	+19.15	+24.50	+37.05	+50.40	+60.10	+72.10	+495.00
1899-1900.....	-22.10	-12.45	-7.90	-5.95	-2.05	+ 0.15	+ 2.05	+ 7.05	+10.30	+31.35	+57.70
1900-1901.....	+ 0.30	+12.80	+18.70	+24.90	+34.80	+43.30	+54.60	+78.60	+110.60	+142.45	+193.00
1901-2.....	+ 1.80	+ 2.60	+ 5.10	+ 6.95	+10.45	+16.05	+21.50	+24.20	+30.85	+40.00	+80.40
1902-3.....	-45.00	-32.90	-20.40	-22.60	-19.60	-17.00	-14.75	-12.75	-	-	+ 1.60
1903-4.....	-26.90	-18.50	-9.60	-5.30	-4.75	-2.10	-0.35	+ 2.05	+ 5.15	+ 9.80	+14.00
1904-5.....	-36.40	+ 5.15	+11.05	+16.30	+22.00	+24.25	+26.75	+32.85	+43.15	+60.35	+94.80
1905-6.....	-11.00	-5.85	-3.35	-2.25	-0.95	+ 3.40	+ 5.20	+ 9.70	+14.70	+27.25	+108.20
1906-7.....	-45.70	-41.40	-37.25	-31.35	-26.65	-22.35	-19.65	-18.00	-12.50	-6.50	+ 5.30
1907-8.....	-30.80	-18.75	-13.55	-6.15	-5.30	-3.35	-1.50	+ 3.85	+ 6.15	+11.15	+25.30
1908-9.....	+ 8.20	+12.35	+16.90	+17.90	+20.30	+30.25	+35.00	+42.15	+49.20	+74.85	+143.20
1909-10.....	-57.00	-30.35	-18.35	-14.60	-10.55	-7.00	-5.90	-0.85	+ 1.60	+ 6.25	+16.00
1910-11.....	-23.80	-21.80	-11.25	-8.35	-6.90	-5.10	-2.75	-1.10	+ 1.30	+ 9.05	+21.90
1911-12.....	-59.70	-22.70	-12.30	-10.30	-5.05	-2.10	-	+ 2.40	+ 6.85	+11.85	+73.80
1912-13.....	-41.50	-35.70	-29.40	-24.80	-19.30	-17.15	-13.55	-11.30	-9.10	-7.15	+ 2.00
1913-14.....	-65.70	-38.80	-31.70	-25.15	-19.75	-14.20	-10.45	-5.85	-4.70	-2.65	+39.00
1914-15.....	-50.00	-21.25	-7.60	-3.60	-2.05	-0.35	+ 2.05	+ 8.05	+19.35	+23.00	+46.70
Averages											
1890-1900....	-38.65	-17.52	-10.38	-5.89	-2.45	+ 1.03	+ 4.64	+ 9.06	+14.29	+28.49	+126.39
1900-1910....	-24.61	-11.49	-5.68	-1.62	+ 2.58	+ 6.55	+10.39	+16.18	+23.98	+36.05	+68.18
1910-1915....	-48.14	-28.05	-18.45	-14.44	-10.71	-7.78	-4.94	-1.56	+ 2.74	+ 6.82	+35.88
1890-1915....	-34.93	-17.21	-10.11	-5.89	-2.09	+ 1.47	+ 5.02	+ 9.78	+15.85	+27.14	+85.00



TABLE XIII

DECILS OF THE RELATIVE PRICES OF FORTY COMMON STOCKS, COMPUTED ON THE BASIS: AVERAGE ACTUAL PRICES IN 1800-99 = 100

Years	Lowest Relative Price	First Decil	Second Decil	Third Decil	Fourth Decil	Median	Sixth Decil	Seventh Decil	Eighth Decil	Ninth Decil	Highest Relative Price
1800.....	30	72	93.5	97.5	105	109.5	120.5	136	148.5	172.5	266
1801.....	30	84.5	94.5	98	102	108	114.5	120.5	132.5	153.5	210
1802.....	70	98	101.5	105	111	115	120	129	142	169.5	210
1803.....	61	66	85.5	89.5	93.5	95	96.5	99	102.5	107.5	147
1804.....	40	52.5	67.5	73.5	79	87.5	91	93	97	101	106
1805.....	30	54.5	71.5	82	87	89	92.5	94.5	99	104	121
1806.....	26	53.5	64	75	77	81.5	85	88.5	92	90	102
1807.....	12	59.5	71	81	85	88.5	90	96.5	98.5	103.5	127
1808.....	17	66	79.5	85.5	89	92.5	98.5	105.5	108.5	124.5	178
1809.....	65	87.5	98	104	109	117	123	138	150.5	173	303
1800.....	60	86	92	102	119	124.5	129	139	151	211	306
1901.....	102	127	137.5	147.5	160.5	188.5	203.5	235.5	287	332.5	488
1902.....	104	139.5	157.5	175	204	220	246.5	288.5	343.5	378.5	564
1903.....	94	119	132.5	158	168.5	183.5	196	205	233.5	298.5	535
1904.....	92	108.5	132.5	159.5	165	172	189	210.5	227	285.5	522
1905.....	97	131	163	194.5	215.5	240.5	254	268.5	313	385.5	689
1906.....	94	127	171.5	200.5	227	238	275.5	292	329	445.5	748
1907.....	63	95.5	123	140	167	179	195.5	224	257	348	610
1908.....	40	92	117	132.5	158	169.5	192	219	259	366.5	586
1909.....	58	118	139	166.5	193.5	209	286	323.5	370.5	517.5	761
1910.....	32	99.5	127.5	139.5	165.5	218.5	233.5	267.5	339	500	817
1911.....	25	91	118	129.5	160.5	188	225	261.5	381	535	878
1912.....	44	86.5	112	120.5	132.5	164.5	202.5	301	384.5	517.5	937
1913.....	33	62	82.5	95.5	113	115	163.5	250	334	461	870
1914.....	14	35.5	65.5	70.5	103.5	112	133.5	240	263	397.5	830
1915.....	7	39.5	73.5	83.5	94	109.5	141	223.5	270.5	392.5	890
Averages											
1800-1899....	38.1	69.4	82.7	89.1	93.8	98.4	103.2	110.1	117.1	130.5	177.0
1900-1909....	81.3	114.4	136.6	157.6	177.8	196.5	216.7	240.6	279.7	356.9	580.9
1910-1915....	25.8	69.0	90.5	106.5	128.2	152.9	183.2	257.3	338.0	467.3	870.3
1800-1915....	51.9	86.6	106.6	119.5	134.0	148.7	165.3	194.2	230.6	295.3	492.4

The only explanation needed concerning them is that the highest and lowest figures for the groups are not averages of the corresponding figures for single stocks, but the extreme figures found after the figures for single stocks have been averaged by years.<sup>1</sup> Tables XI, XII, and XIII correspond to the preceding set in showing respectively actual prices, ratios of change from one year to the next, and relative prices computed on a fixed base. They differ from Tables VIII–X in not giving the names of particular stocks, but in showing for each year the highest and lowest figures and the nine intervening decils (of which the fifth is the median)—that is, the points which divide into ten equal parts the forty entries for each year, after these entries have been arranged in order of magnitude from lowest to highest.<sup>2</sup>

At this point the reader should surrender himself to the fascination of these six tables for a time, and not return to the text until the questions which the figures suggest and do not answer begin to clamor for discussion.

With many of the facts regarding particular stocks which make these tables interesting to investors the maker of index numbers is little concerned. What does concern him deeply is the wide dispersion of stock prices and of their variations, whether these variations be counted in dollars and cents, in percentages of yearly rise and fall, or in relative prices made on a fixed base. For it is this wide dispersion that accounts for the divergences among different index numbers made from the same data, and that raises

<sup>1</sup> For example, Table VIII, column 5, shows that in the North Atlantic group one stock reached its highest yearly price in 1890, two stocks in 1902, and two in 1905. The average of these individual maximum prices would be \$133.80. But when the yearly prices of the five stocks are treated as a group, the highest average price occurs in 1902, and that price is only \$122.58—the entry in the table.

<sup>2</sup> Of these tables, the only one hitherto published even in part is Table XIII. The present version of this table differs from its predecessors (*Journal of Political Economy*, XVIII, 375; *Business Cycles*, p. 192) in one particular. Some statisticians decline to recognize as a true median the computed value which lies midmost between the two middle members of a series having an equal number of terms. Of course the same scruple applies in locating the decils. Hence I formerly adopted the arbitrary rule of not counting the highest figure in finding the decils—a rule which left thirty-nine entries and gave each decil a definite location. In the present case, however, I have counted in the full forty stocks, and accepted at their face value computed figures intermediate between two entries.

a serious doubt whether any of these index numbers possess much significance.

This difficulty is much more serious in dealing with stocks than in dealing with commodities. Table XIV summarizes the results

TABLE XIV

COMPARISON OF THE PERCENTAGE VARIATIONS IN THE PRICES OF STOCKS AND OF COMMODITIES AT WHOLESALE FROM ONE YEAR TO THE NEXT

Averages are Arithmetic Means

	Forty Common Stocks 1890-1915	Forty Selected Commodities 1890-1914	All Commodities Quoted by B.L.S. in 1890-1913
	Per Cent	Per Cent	Per Cent
Average change.....	20.1	13.0	8.6
Average change in cases of fall.....	14.8	11.6	8.8
Average change in cases of rise.....	26.9	14.4	10.9
Average net rise in period covered.....	4.4	2.0	1.4
Greatest rise of any item in any year.....	495.0	103.3	103.3
Greatest fall of any item in any year.....	78.4	54.6	54.6

of a comparison between the percentage variations in stock and commodity prices from one year to the next. Against data concerning stocks drawn from Tables IX and XII it sets similar figures for all commodities quoted by the Bureau of Labor Statistics in 1890-1913 and for forty commodities selected from the Bureau's list because of their exceptional variability in price.<sup>1</sup>

<sup>1</sup> The figures for all commodities quoted by the Bureau are taken from the data used in making Tables II and IV of *Bulletin No. 173*, pp. 15 and 19. The figures for forty selected commodities are from a new compilation based upon *Bulletin No. 181*, Table II. The commodities in question are the following:

- |                                   |   |
|-----------------------------------|---|
| 1. Cotton, middling, New York     | 21. Wool, Ohio, medium                            |
| 2. Corn, cash contract            | 22. Coal, Georges Creek, at mine                  |
| 3. Oats, cash                     | 23. Coke  |
| 4. Wheat, cash, Chicago           | 24. Petroleum, crude                              |
| 5. Hides, green salted            | 25. Copper, ingot                                 |
| 6. Cattle, steers, good to choice | 26. Lead, pig                                     |
| 7. Hogs, heavy                    | 27. Pig iron, foundry, No. 2                      |
| 8. Sheep, western                 | 28. Steel billets                                 |
| 9. Butter, creamery, New York     | 29. Tin, pig                                      |
| 10. Coffee, Rio                   | 30. Spelter                                       |
| 11. Eggs, New York                | 31. Brick   |
| 12. Fish, cod                     | 32. Cement, Rosendale 1890-99, domestic 1900-1914 |
| 13. Flour, wheat, standard        | 33. Linseed oil, raw                              |
| 14. Meal, corn, yellow            | 34. Lumber, spruce                                |
| 15. Salt, American                | 35. Rosin   |
| 16. Pepper                        | 36. Tar   |
| 17. Sugar, 96° centrifugal        | 37. Quinine                                       |
| 18. Onions                        | 38. Jute  |
| 19. Potatoes, white               | 39. Paper, news                                   |
| 20. Silk, raw, Japanese           | 40. Rubber  |

More significant still for present purposes are the next two tables which put averages derived from the preceding tables of decils against averages derived from similar decils of the relative prices of commodities.<sup>1</sup> They represent the average annual range covered by the price fluctuations of stocks and commodities when these fluctuations are counted as percentages of prices in the preceding year (Table XV) and as percentages of prices in 1890-99 (Table XVI). In all four distributions there is a marked concentration toward the center of the field; that is, the inner tenths are packed into a much narrower range than the outer tenths. But this concentration is much less dense for stocks than for commodities; it is much more dense for both commodities and stocks when fluctuations are reckoned on the preceding-year base than when they are reckoned on the 1890-99 base; and the difference between the densities of concentration on these two bases is much wider for stocks than for commodities.

TABLE XV

AVERAGE ANNUAL DISPERSION OF THE RELATIVE PRICES OF STOCKS AND COMMODITIES  
COMPUTED ON THE BASIS: PRICES IN THE PRECEDING YEAR=100

Average Number of Points on the Percentage Scale between the	Stocks	Commodities
Lowest and highest relative prices	119.9	95.1
First and ninth decils. . . . .	44.4	25.7
Second and eighth decils. . . . .	26.0	13.9
Third and seventh decils. . . . .	15.7	7.8
Fourth and sixth decils. . . . .	7.1	3.6
Lowest relative prices and first decils. . . . .	17.7	27.0
First and second decils. . . . .	7.1	4.9
Second and third decils. . . . .	4.2	2.6
Third and fourth decils. . . . .	3.8	2.2
Fourth decils and medians. . . . .	3.6	1.8
Highest relative prices and ninth decils. . . . .	57.9	42.4
Ninth and eighth decils. . . . .	11.3	6.9
Eighth and seventh decils. . . . .	6.1	3.5
Seventh and sixth decils. . . . .	4.8	2.0
Sixth decils and medians. . . . .	3.5	1.8

<sup>1</sup> For the decils of commodity prices see *Bulletin of the U.S. Bureau of Labor Statistics No. 173*, p. 17, and *Business Cycles*, p. 112. Both tables (the latter after extension) cover the years 1890-1913. The first includes all commodities quoted by the Bureau of Labor Statistics; the latter includes the 145 commodities used in the writer's revised form of the Bureau's index. For this list see *Bulletin No. 173*, p. 47, note.

TABLE XVI

AVERAGE ANNUAL DISPERSION OF THE RELATIVE PRICES OF STOCKS AND COMMODITIES  
COMPUTED ON THE BASIS: AVERAGE PRICES IN 1890-99 = 100

Average Number of Points on the Percentage Scale between the	Stocks	Commodities
Lowest and highest relative prices	440.5	170.9
First and ninth decils. . . . .	208.7	53.7
Second and eighth decils. . . . .	124.0	30.5
Third and seventh decils. . . . .	74.7	18.1
Fourth and sixth decils. . . . .	31.3	8.3
Lowest relative prices and first decils. . . . .	34.7	33.8
First and second decils. . . . .	20.0	8.7
Second and third decils. . . . .	12.9	5.7
Third and fourth decils. . . . .	14.5	4.6
Fourth decils and medians. . . . .	14.7	4.3
Highest relative prices and ninth decils. . . . .	197.1	83.4
Ninth and eighth decils. . . . .	64.7	14.5
Eighth and seventh decils. . . . .	36.4	6.7
Seventh and sixth decils. . . . .	28.9	5.2
Sixth decils and medians. . . . .	16.6	4.0

In just one respect stock fluctuations are more uniform than commodity fluctuations. Table XII shows that in two years since 1890 every stock in our representative list rose in price (1900-1901 and 1908-9), and that in one year every stock fell (1912-13). Such a thing seems never to happen in the commodity markets. There was no year in the period 1890-1913 when at least 8 per cent of the 240 commodities (more or less) quoted by the Bureau of Labor Statistics did not rise, when at least 16 per cent did not fall, and when at least 8 per cent did not remain unchanged in price. It is this greater uniformity in the general trend of stock fluctuations that explains the fact pointed out above, that stock index numbers agree rather better among themselves than do commodity indexes regarding the direction of the fluctuations, though they differ more regarding the degree.

The great obstacle in the way of measuring fluctuations in the prices of stocks, then, is their extreme diversity. Of course the representative value of averages is less when the data averaged are widely and irregularly scattered than when they are compactly and symmetrically grouped. Our problem, therefore, turns into the

question whether the data concerning changes in stock prices can be arranged in any fashion that gives a fairly compact and symmetrical grouping.

VII. ACTUAL PRICES, PERCENTAGES OF CHANGE FROM ONE YEAR TO THE NEXT, AND RELATIVE PRICES ON A FIXED BASE

One phase of this problem is to determine which of our three sets of measures—actual prices, percentages of change in price from one year to the next, or relative prices on the 1890-99 base—has the most compact and regular grouping of its constituent items. Some light has been thrown upon this question by Tables XIV-XVI; but we may settle it definitely by computing the percentage deviations of the individual entries in each set of measures from their respective arithmetic means and seeing in which case the deviations are least for the whole period of twenty-six years covered by the tables. Indeed it is not necessary to take each of the forty entries for each year; a computation limited to the materials presented in Tables XI-XIII—the highest and lowest items and the nine decils—will suffice.

The results, assembled in Table XVII, leave no room for doubt. Percentages of change from prices in the preceding year are much less scattered than actual prices or fixed-base relatives. That fact justifies the conclusion that stock index numbers made by averaging percentages of yearly change in prices are more reliable measures than are those made by averaging actual prices or fixed-base relatives.

What is less to be expected, the 1890-99 relative prices are distinctly less scattered than the actual prices. Therefore the index numbers in Table III, which are made by averaging relative prices (Nos. 1, 2, and 9), are more reliable than their fellows made from averages of actual prices, or the latter averages themselves, as shown in Table I. But these fixed-base relatives have a special defect: their degree of dispersion grows greater as we get farther and farther from the years that form the base. In 1910-15 they no longer have a decisive advantage over the actual prices, and as time goes on they may well have a decisive disadvantage. Table XVII shows that the other measures have no such inherent tendency

TABLE XVII

AVERAGE DEVIATIONS OF THE ACTUAL PRICES OF FORTY COMMON STOCKS, THEIR RELATIVE PRICES ON THE PRECEDING-YEAR BASE, AND THEIR RELATIVE PRICES ON THE 1890-99 BASE, FROM THE RESPECTIVE ARITHMETIC MEANS:

ARITHMETIC MEANS MADE FROM DATA FOR EACH OF THE YEARS 1890-1915

(Based upon Tables XI-XIII and the corresponding arithmetic means in Tables I-III)

PERCENTAGES BY WHICH THE LOWEST AND HIGHEST ENTRIES AND THE DECILS WERE LESS (-) OR GREATER (+) THAN THE CORRESPONDING ARITHMETIC MEANS											
Grand Averages 1890-1915			Relative Prices on Preceding-Year Base			Relative Prices on 1890-99 Base			Actual Prices		
Relative Prices on Preceding-Year Base (1)	Relative Prices on 1890-99 Base (2)	Actual Prices (3)	1890-91 to 1899-1900 (4)	1900-1901 to 1909-10 (5)	1910-11 to 1914-15 (6)	1890 to 1899 (7)	1900 to 1909 (8)	1910 to 1915 (9)	1890 to 1899 (10)	1900 to 1909 (11)	1910 to 1915 (12)
Lowest.....	- 38	- 91	- 42	- 31	- 43	- 62	- 62	- 88	- 91	- 88	- 96
First decil.....	- 21	- 82	- 22	- 19	- 22	- 31	- 47	- 69	- 83	- 77	- 88
Second decil.....	- 13	- 73	- 14	- 13	- 11	- 17	- 37	- 56	- 75	- 68	- 77
Third decil.....	- 9	- 63	- 10	- 9	- 7	- 10	- 27	- 52	- 68	- 58	- 63
Fourth decil.....	- 5	- 47	- 7	- 6	- 2	- 5	- 18	- 42	- 58	- 39	- 42
Median.....	- 2	- 27	- 3	- 2	+ 1	- 1	- 10	- 31	- 43	- 19	- 13
Sixth decil.....	+ 1	+ 1	0	+ 1	+ 1	+ 4	- 1	- 17	- 6	+ 3	+ 9
Seventh decil.....	+ 5	+ 35	+ 4	+ 6	+ 8	+ 10	+ 10	+ 19	+ 40	+ 35	+ 27
Eighth decil.....	+ 11	+ 69	+ 9	+ 12	+ 12	+ 17	+ 27	+ 55	+ 93	+ 60	+ 47
Ninth decil.....	+ 21	+ 103	+ 22	+ 22	+ 17	+ 29	+ 63	+ 115	+ 123	+ 97	+ 80
Highest.....	+ 70	+ 305	+ 103	+ 49	+ 48	+ 72	+ 165	+ 307	+ 280	+ 273	+ 399
Range between first and ninth decils..	42	185	44	41	39	60	110	184	206	174	168

to scatter with the lapse of time. Of course this fact explains why the index numbers on the 1890-99 base differ more from each other in 1910-15 than in 1900-1909, and in 1900-1909 than in 1890-99 (Table VI).<sup>1</sup>

#### VIII. ARITHMETIC MEANS, GEOMETRIC MEANS, AND MEDIANS

In one respect even the best groupings of Table XVII are asymmetrical: the average deviations from the arithmetic means run farther up the percentage scale than they run down. Similarly, in Tables XI-XIII the margins between the medians and the ninth decils are wider than the margins between the medians and the first decils. A method of grouping the data which produced a better balance on either side of the average would make the average itself a more representative figure.

The obvious experiment to try in seeking a better-balanced distribution is a regrouping along a geometric instead of an arithmetic scale; that is, we may treat an entry which equals twice the arithmetic mean of its series as balanced by another entry which equals half the arithmetic mean, etc. On the arithmetic scales used so far these entries would not balance—one would be 100 points above the mean, the other 50 points below. The deviations of the highest and lowest entries and the nine decils of Tables XI-XIII from their respective arithmetic means have already been computed for each year as materials for Table XVII, and we have only to rearrange them in new form to try this experiment. Instead of making twenty-six-year averages we make frequency tables—that is, tables showing the number of times deviations of each given magnitude occur. Then we draw up a geometric scale

<sup>1</sup> In a footnote to the last paragraph of Section III it was pointed out that relative prices are merely actual prices altered by the application of certain multipliers or divisors—that is, they are weighted actual prices. The conclusions of this section may therefore be rephrased as follows: (1) the prices of stocks are more compactly grouped and therefore yield more significant averages when they are weighted by the factors required to make the price of each stock in the preceding year equal 100 than when they are weighted by the factors required to make the prices of each stock in some fixed period equal 100; and (2) the latter weighting in turn yields better groupings and therefore more significant averages than the unweighted actual prices so long as the period selected as base lies not too long in the past. If the equivalence of these statements and those made in the text is not at once clear, the reader may turn for explanations to Section X.



of deviations from the arithmetic means by choosing a convenient series of numbers to be used both as multipliers and as divisors. When used as multipliers, these numbers give values greater than the arithmetic mean—and these values constitute our scale of deviations above the mean. When used as divisors, they give a corresponding scale of deviations below the means. Then we have only to count and enter the number of deviations in our frequency table falling within the successive intervals marked by our geometric scale. Table XVIII gives the finished results.<sup>1</sup>

The experiment is an unqualified success so far as relates to percentages of change from one year to the next. They are admirably balanced in this table—much better balanced than on an arithmetic scale. For these fluctuations, then, the geometric mean is a more significant average than the arithmetic mean. The same conclusion may be drawn, though with less emphasis, regarding the 1890-99 relatives. They balance better on a geometric than on an arithmetic scale; but they balance less evenly on either scale than do the relatives on the preceding-year base. As for the actual prices, it cannot be said that they are symmetrically grouped on either scale. The geometric distribution may not be quite so bad as the other, but neither distribution is good. Index numbers made by averaging actual prices of stocks, therefore, have less representative value than those made by averaging relative prices—always excepting, for reasons given above, relative prices computed on a fixed base more than, say, ten years past.<sup>2</sup>

It is interesting to compare the best of our distributions of data for stocks with the distribution of corresponding data for commodities. The latter data are ready to hand, not in the form of

<sup>1</sup> Compare the experiment tried and the conclusion drawn by Mr. F. R. Macaulay with reference to decils of relative prices of commodities on the preceding-year base (one of the tables in *Bulletin of the U.S. Bureau of Labor Statistics*, No. 173) (*American Economic Review*, VI, 205-7 [March, 1916]).

<sup>2</sup> Even so, geometric means of actual prices have one special advantage: they can be turned at once into geometric means of relative prices on any base that is desired—the preceding year, 1890-99, 1890-1915, the last completed year, etc. As was shown in Section V above, the arithmetic means or medians of actual prices, on the contrary, do not yield arithmetic means or medians of relative prices: if the latter are required, the relative prices must be computed and averaged or an equivalent set of weights must be applied to the actual prices. If an explanation of this difference is wanted, see "The Making and Using of Index Numbers," pp. 83-86.

TABLE XVIII

GEOMETRIC DISTRIBUTION OF DEVIATIONS OF THE ACTUAL AND RELATIVE PRICES OF FORTY COMMON STOCKS IN THE YEARS 1890-1915  
FROM THEIR RESPECTIVE ARITHMETIC MEANS

(Based upon the highest and lowest figures and the nine decils of actual prices, relative prices on the preceding-year base, and relative prices on the 1890-99 base, as shown in Tables XI-XIII, and upon the corresponding arithmetic means, as shown in Tables I-III)

SERIES OF MULTIPLIERS AND DIVISORS	RESULTING SCALES OF PERCENTAGE DEVIATIONS FROM THE ARITHMETIC MEANS		RELATIVE PRICES ON THE PRECEDING-YEAR BASE		RELATIVE PRICES ON THE 1890-99 BASE		ACTUAL PRICES	
	(2) Deviations below the Arithmetic Means	(3) Deviations above the Arithmetic Means	(4) No. of Devia- tions below the Arithmetic Means	(5) No. of Devia- tions above the Arithmetic Means	(6) No. of Devia- tions below the Arithmetic Means	(7) No. of Devia- tions above the Arithmetic Means	(8) No. of Devia- tions below the Arithmetic Means	(9) No. of Devia- tions above the Arithmetic Means
(1)								
1.0-1.00.....	99.9-01.0	100-100	66	60	27	29	4	11
1.1-1.10.....	90.9-83.4	110-110	33	29	28	25	15	5
1.2-1.20.....	83.3-77.0	120-129	21	14	13	14	6	7
1.3-1.39.....	76.9-71.5	130-139	13	6	8	9	3	14
1.4-1.49.....	71.4-66.8	140-149	4	1	10	5	1	7
1.5-1.59.....	66.7-62.6	150-159	1	4	13	10	4	7
1.6-1.69.....	62.5-58.9	160-169	1	1	7	2	9	6
1.7-1.79.....	58.8-55.7	170-179	1	3	5	2	2	6
1.8-1.89.....	55.0-52.7	180-189	2	1	3	4	3	8
1.9-1.99.....	52.6-50.1	190-199	1	1	7	1	4	5
2.0-2.10.....	50.0-45.6	200-219	2	4	11	3	7	12
2.2-2.29.....	45.5-41.8	220-239	1	1	3	7	4	6
2.4-2.50.....	41.7-38.6	240-259	2	.....	.....	.....	4	2
2.6-2.79.....	38.5-35.8	260-279	.....	.....	5	4	10	1
2.8-2.99.....	35.7-33.4	280-299	.....	.....	3	3	9	.....
3.0-3.99.....	33.3-25.1	300-399	.....	1	4	3	24	10
4.0-4.99.....	35.0-20.1	400-499	1	.....	4	3	12	8
5.0-5.99.....	20.0-16.8	500-599	.....	.....	2	.....	9	5
6.0-6.99.....	16.7-14.4	600-699	.....	.....	1	.....	9	.....
7.0-7.99.....	14.3-12.6	700-799	.....	.....	2	.....	3	.....
8.0-8.99.....	12.5-11.2	800-899	.....	.....	.....	.....	1	.....
9.0-9.99.....	11.1-10.1	900-999	.....	.....	.....	.....	2	.....
10.0-10.99.....	10.0-5.1	1,000-1,999	.....	.....	2	.....	13	.....
20.0-20.99.....	5.0-3.4	2,000-2,999	.....	.....	1	.....	6	.....
30.0-49.99.....	3.3-2.1	3,000-4,999	.....	.....	.....	.....	.....	.....
50.0-99.99.....	2.0-1.9	5,000-9,999	.....	.....	.....	.....	1	.....
100.0.....	1.0 and less	10,000 and more	.....	.....	.....	.....	1	.....
Sums.....	.....	.....	149	126	162	124	166	120
			275		286		286	

deviations from arithmetic means, but in the form of percentages of yearly rise or fall in price. However, these percentages can readily be distributed on a geometric scale and compared with the similar percentages of change in the prices of stocks. Table XIX brings the two sets of figures together. In both cases the grouping is highly symmetrical, more symmetrical than the groupings on the arithmetic scale; hence the geometric mean is technically a better average than the arithmetic mean for year-to-year fluctuations in the prices of commodities as well as for fluctuations in the prices of stocks. Further, the commodity fluctuations are more highly concentrated in the center of the field than stock fluctuations, and their extremes cover a narrower range, so that with equally representative samples to use, the index numbers of commodity prices have higher representative value than the index numbers of stock prices.

So far the argument runs: geometric means are more significant averages of price fluctuations, whether of commodities or stocks, than arithmetic means, because they are the averages of more symmetrical distributions. What, then, of medians? They have high representative value when the distribution shows concentration about the middle members of the series, and when the scattering cases of wide deviation on either side of the middle possess little significance. Tables XI-XIII show that the arithmetic means of the distributions under discussion must be unduly influenced by a few extremely high actual or relative prices. On the other hand, Table XVIII shows that the geometric means of actual prices, and in less measure of the 1890-99 relatives, are in danger of distortion from the occurrence of numerous exceptionally low values, not counterbalanced by offsetting high values. Does it follow, then, that medians are better averages of these data than either of their rivals?

Certainly we found in Section IV that the index numbers based upon medians of actual prices (column 4, Tables II and III) are highly erratic. On the average they jump more from one year to the next than any of the other series, and on the average they show the widest disagreement with the other series (Table V). Moreover, in one year this series contradicts the evidence of ten other series, indicating that stock prices advanced in 1910, while all the others say that stocks fell. When one looks back to the figures

from which the medians are taken, the drop in this year seems most unrepresentative of the general trend. After the actual prices of all forty stocks in 1909 and 1910 have been ranged in order of magnitude, it appears that in thirty-four cases the 1910 entries are lower than those of the preceding year and in only six cases higher; but two of these six cases happen to fall in the center of the series. The fact is that the actual prices show little of that concentration about the middle members of the series which is required to give the medians stability and significance. Instead of concentration there is often a wide gap between the twentieth and the twenty-first entries—a gap which often exceeds \$5.00 per share, and once exceeds \$25.00. The odds are that the addition to or subtraction from our list of one stock taken at random would make a greater difference in the medians than in the arithmetic or the geometric means.

Similar defects, though in lesser degree, beset the medians of relatives on the 1890-99 base. This series (Table III, No. 1) differs from all the others by saying that stock prices fell in the highly prosperous year 1906. On turning to the full array of these relatives for 1905 and 1906 we find that only ten of the forty entries arranged in numerical order show a decline; but two of these ten happen to be in the strategic position of fixing the median.

To conclude: If quotations could be found for two or three times the number of stocks employed here, it is probable that the distribution of the data would be rendered more regular. Then the actual prices might be grouped about their central tendency with sufficient symmetry to yield averages of high representative value. Even the medians might lose that erratic character which they now have and become excellent averages. But with only forty stocks as samples it must be confessed that averages of actual prices, whatever mean be tried, are open to question.<sup>1</sup> One

<sup>1</sup> This remark concerning the series including forty stocks applies a fortiori to the *Wall Street Journal's* series for twenty railways, to the *Annalist's* averages for twenty-five railways, and to other index numbers of stocks built upon an even narrower foundation. It is quite possible that the actual prices from which these smaller series are made show a less irregular grouping than my forty stocks. But to accomplish that result there must be arbitrary selection of the samples. If the data are made more uniform by that means, they become at the same time less representative of the whole situation.

TABLE XIX

GEOMETRIC DISTRIBUTION OF THE RELATIVE PRICES OF STOCKS AND COMMODITIES ON THE PRECEDING-YEAR BASE  
 (Made from the data used for Table XII above [forty stocks, 1890-1915], and for Tables 2 and 4, *Bulletin of the U.S. Bureau of Labor Statistics, No. 173* [240± commodities, 1890-1913])

SERIES OF MULTIPLIERS AND DIVISORS	SCALES OF RELATIVE PRICES ON PRECEDING-YEAR BASE RESULTING FROM USE OF THE SERIES OF MULTIPLIERS AND DIVISORS		NUMBER OF TIMES PER MILE EACH RELATIVE PRICE OCCURRED					
	Falling Prices (2)	Rising Prices (3)	Falling Prices		No Change in Price		Rising Prices	
			Commodities (4)	Stocks (5)	Commodities (6)	Stocks (7)	Stocks (8)	Commodities (9)
1.00.....	99.9-91.0	100.1-109	263	211	125	6	180	290
1.001-1.09.....	99.9-83.4	110-119	92	129	104	104	104	101
1.10-1.19.....	83.3-77.0	120-129	32	69	68	68	68	39
1.20-1.29.....	76.9-71.5	130-139	14	39	34	34	34	15
1.30-1.39.....	71.4-66.8	140-149	7	21	21	21	21	8
1.40-1.49.....	66.7-62.6	150-159	3	18	17	17	17	3
1.50-1.59.....	62.5-58.9	160-169	2	11	11	11	11	2
1.60-1.69.....	58.8-55.7	170-179	1.3	8	7	7	7	1
1.70-1.79.....	55.6-52.7	180-189	.4	2	5	5	5	.6
1.80-1.89.....	52.6-50.1	190-199	.2	2	5	5	5	.4
1.90-1.99.....	50.0-45.6	200-219	.1	2	6	6	6	.....
2.00-2.19.....	45.5-41.8	220-239	.....	2	.....	.....	.....	.....
2.20-2.39.....	41.7-38.6	240-259	.....	1	.....	.....	.....	.....
2.40-2.59.....	38.5-35.8	260-279	.....	.....	.....	.....	.....	.....
2.60-2.79.....	35.7-33.4	280-299	.....	2	.....	.....	.....	.....
2.80-2.99.....	33.3-25.1	300-399	.....	.....	.....	.....	.....	.....
3.00-3.99.....	25.0-20.1	400-499	.....	1	.....	.....	.....	.....
4.00-4.99.....	20.0-16.8	500-599	.....	.....	.....	.....	.....	.....
5.00-5.99.....	.....	.....	.....	.....	.....	.....	.....	.....
Totals.....	.....	.....	415.0	523	125	6	471	460.0

SUMMARY

	TOTAL NO. OF CASES OBSERVED	NO. OF CASES IN WHICH PRICES			NO. OF CASES PER MILE IN WHICH PRICES			MEDIAN OF ALL CASES	ARITHMETIC MEANS OF ALL CASES
		Fell	Did Not Change	Rose	Fell	Did Not Change	Rose		
Stocks.....	1,000	523	6	471	523	6	471	99.05	104.35
Commodities.....	5,578	2,314	697	2,567	415	125	400	100.00	101.30

set of fluctuations, however, can be measured rather accurately—even when no more than forty stocks are included—namely, the average percentages of change in price from one year to the next. For such measurements the geometric mean has been shown to be technically the most significant average. Further, it seems fair to say that relative prices on a fixed base yield acceptable (though less excellent) averages with a sample no larger than forty, so long as the computation is not carried many years beyond the base period.<sup>1</sup> In averaging these fixed-base relatives, also, the geometric mean is technically preferable to the arithmetic mean or median.

#### IX. THE KINDS OF STOCKS INCLUDED

Students of commodity index numbers have found that the various groups into which commodities are divided by conditions of supply and demand exhibit characteristically different types of fluctuation in prices. Raw materials as a class vary more in price than manufactured goods. Among raw materials the prices of mineral products depend largely upon the activity of business; the prices of farm crops and, in a somewhat different way, the prices of animal products depend largely upon the weather, while the prices of forest products in America at present show the influence of the progressive reduction of natural resources. Among manufactured articles staple consumers' goods are steadier in price than staple producers' goods, trust-made goods are steadier than strictly competitive products, etc.<sup>2</sup>

In making and in using commodity index numbers it is highly important to recognize the existence and the characteristics of these groups, for the differences among them are so wide as to determine the behavior of the series into which they enter in different proportions. The ideal procedure is to include enough commodities of each group to secure a fair sample, to make for each group a

<sup>1</sup> If the fixed-base relatives in the preceding tables were recomputed on the 1890-1915 base, it is probable that the figures would show more regular distribution than the 1890-99 relatives, to say nothing of the actual prices. But even these new relatives would probably become badly scattered and therefore of questionable value after the lapse of ten or fifteen years more.

<sup>2</sup> All these propositions, except the last, are demonstrated in "The Making and Using of Index Numbers," *Bulletin of the U.S. Bureau of Labor Statistics*, No. 173, pp. 52-71.

separate series of index numbers, and then to combine all of these series in a properly weighted average to ascertain the changes in the wholesale price level at large.

Although we found in Section V that the sample used in a stock index number makes less difference in the results than is made by the form of average chosen, the scheme of weights applied, or the stage at which actual prices are reduced to relatives, still it is proper to inquire whether there are among stocks, as among commodities, distinct groups which demand recognition in making index numbers.

Stocks are of course classified in many different ways, and for several of the resulting groups separate index numbers have been computed. (1) The *Wall Street Journal* and other financial papers publish independent series for railway stocks and "industrials." The chief difference between these index numbers is that the industrials vary even more in price than the railways.<sup>1</sup> This distinction, based on the nature of the business transacted, is clearly important and deserves to be developed further whenever the data permit: separate index numbers for the stocks of, say, mining and smelting companies, iron and steel companies, machinery and equipment concerns, textile factories, public utilities, etc., would prove highly significant if enough quotations could be had to make trustworthy averages for each group.

My own figures have been confined to the stocks of transportation companies, not from choice, but because few other stocks have been regularly quoted throughout the rather long period I started to study. But among these transportation stocks I have found several interesting groups, for which index numbers are assembled in Table XX. It will be noticed that the number of

<sup>1</sup> A comparison between annual averages struck from the *Wall Street Journal's* monthly highest and lowest figures for twenty railway and for twelve industrial stocks in 1897-1914 shows the average change from one year to the next to be 12.1 per cent for the first group and 14.3 per cent for the second. On the other hand, the net change between 1897 and 1914 is a rise of 73 per cent for railways and of 70 per cent for industrials. This latter result, however, may be quite untrustworthy, not only because of the above-demonstrated difficulty of making long-period comparisons of stock prices, but also because of changes made from time to time in the *Journal's* lists of stocks, the small number of securities included, and the doubtful value of arithmetic means of actual prices. In three years out of eighteen (this time including 1915) the two series have moved in opposite directions.



TABLE XX

INDEX NUMBERS SHOWING THE DIVERSITY OF FLUCTUATIONS AMONG DIFFERENT GROUPS OF STOCKS. ARITHMETIC MEANS OF RELATIVE PRICES COMPUTED ON THE 1890-99 BASE

TEN STOCKS PAYING DIVIDENDS EVERY YEAR	PREFERRED AND COMMON STOCKS IN THE SAME TEN RAILWAYS		COMMON STOCKS CLASSIFIED ACCORDING TO AVERAGE ACTUAL PRICES IN 1890-99					COMMON STOCKS CLASSIFIED BY CHARACTER OF BUSINESS AND TERRITORY SERVED				
	Preferred	Common	17 Stocks Worth Less than \$35 per Share	13 Stocks Worth \$45-\$99 per Share	10 Stocks Over \$100 per Share	5 North Atlantic Railways	4 Anthra- cite Coal- carrying Railways	6 Southern Railways	8 Middle- western Railways	7 North- western Railways	5 Pacific Railways	5 Express, Steam- ship, and Tele- graph Com- panies (13)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1890.....	108	113	125	127	106	113	117	132	120	105	144	117
1891.....	100	113	118	115	101	108	106	123	118	94	134	108
1892.....	115	124	138	126	107	119	135	114	128	119	139	108
1893.....	87	89	92	92	99	97	97	84	93	92	99	96
1894.....	82	82	76	77	95	89	95	78	83	76	95	89
1895.....	87	87	85	79	95	89	89	82	93	85	68	91
1896.....	85	81	73	72	91	86	88	67	77	78	64	85
1897.....	94	90	78	84	93	90	89	89	76	88	67	91
1898.....	107	97	89	98	100	95	84	98	93	100	87	99
1899.....	130	126	136	129	114	115	99	134	120	164	132	117
1900.....	133	144	147	130	116	110	99	151	123	180	145	108
1901.....	178	262	271	186	140	158	144	259	206	267	259	137
1902.....	101	305	325	221	161	162	181	297	256	332	292	171
1903.....	163	238	240	187	141	133	161	259	188	245	236	151
1904.....	156	215	221	192	142	130	170	250	174	214	240	153
1905.....	181	278	297	248	173	186	255	325	214	281	306	176
1906.....	180	291	319	265	183	177	305	341	227	289	363	178
1907.....	138	206	225	214	155	127	245	269	153	190	320	152
1908.....	130	195	218	214	157	125	270	246	150	192	326	142
1909.....	169	280	332	271	193	159	340	358	201	296	445	177
1910.....	148	248	301	260	166	141	331	349	197	243	406	148

1911.....	137	233	296	253	160	140	328	345	178	244	405	137
1912.....	141	215	278	257	165	138	364	347	173	210	397	130
1913.....	121	184	234	224	135	115	320	295	147	175	373	91
1914.....	107	84	190	206	122	96	304	256	111	149	309	85
1915.....	109	81	162	200	127	109	310	251	111	150	305	91
Averages												
1890-1899.....	100	100	100	100	100	100	100	100	100	100	100	100
1900-1909.....	141	162	241	213	156	147	217	276	189	249	293	155
1910-1915.....	126	113	200	233	146	123	326	307	153	195	363	114
1890-1915.....	122	127	177	174	132	123	197	215	147	179	235	124
Net rise (+) or fall (-)												
1890-1896.....	-12	-23	-52	-55	-15	-27	-29	-65	-43	-27	-80	-32
1896-1909.....	+58	+84	+259	+199	+102	+73	+252	+291	+124	+218	+381	+92
1909-1915.....	-41	-88	-135	-71	-66	-50	-30	-107	-90	-146	-140	-86
1890-1915.....	+5	-27	+49	+73	+21	-4	+193	+119	-9	+45	+161	-26
Difference between highest and lowest.....												
Average change from one year to the next.....	67	110	224	199	102	100	280	291	180	256	381	93
	9.7	16.0	30.7	21.2	12.4	15.8	24.9	26.4	24.7	35.9	32.6	14.2

NOTE.—The entries italicized in each series are the highest and lowest within the period covered.

stocks included in at least half of these groups is so small that the averages have no great value. (2) Stocks which pay dividends regularly are steadier in price both from year to year and over long periods of time than other stocks.<sup>1</sup> The summaries of column 1 of the table present a marked contrast to those of the other columns. (3) As a group preferred stocks are steadier than common stocks in the same railways. (4) Stocks that were already high-priced in 1890-99 have varied less from year to year and risen less since then than stocks that were then low-priced.<sup>2</sup> (5) The geographical groups of columns 7-12 represent many striking divergences which it is needless to write out in detail. (6) Sensational differences would appear if separate index numbers were computed for the "Harriman stocks," the "Vanderbilt stocks," the "Gould stocks," etc., as one may judge by looking back at Tables VIII and X. (7) The stocks largely traded in on the Stock Exchange behave somewhat differently from the less active list.<sup>3</sup> (8) Other contrasts would probably appear if the

<sup>1</sup> For the names of the ten dividend-paying stocks see *Business Cycles*, p. 203, n. 76.

<sup>2</sup> A similar comparison based upon average actual prices for the whole period 1890-1915 yields the following summary:

	PERCENTAGE OF CHANGE IN PRICE FROM ONE YEAR TO THE NEXT		
	Average	Highest for Any Stock	Lowest for Any Stock
	Per Cent	Per Cent	Per Cent
Fourteen stocks having average prices above \$100 in 1890-1915.....	11.4	16.8	6.1
Twelve stocks having average prices of \$40-\$99 in 1890-1915.....	20.0	28.4	7.7
Fourteen stocks having average prices below \$40 in 1890-1915.....	28.8	45.0	18.2

The names of the stocks included in these three groups may be made out from Table VIII, column 1.

<sup>3</sup> I have carried this comparison far enough to make out that the most active stocks in 1890-1914 as a group have varied less in price from one year to the next than other groups in which transactions were smaller. However, the following summary which establishes this result raises more questions than it answers. It will be noticed that *within* the groups there is some evidence of correspondence between amount of sales and variability in price. Perhaps a more instructive comparison could be made by taking ratios between sales and amount of stock outstanding as the basis for com-

railways that have increased their stock issues largely were segregated from those that have not; (g) if the railways that have and have not undergone reorganization within the period covered were separately treated, etc.

One might go on at considerable length with this catalogue of groups; but the list as it stands is long enough to bring out the significant conclusions. (1) Such groups are important for the

parison. Or the inquiry might be elaborated by taking each year by itself and comparing ratios of change with amounts or ratios of sales.

	Thousands of Shares Sold Yearly; Averages for 1890-1914	Average Change in Price from One Year to the Next
		Per Cent
Union Pacific.....	11,069	28.4
Reading.....	6,891	28.0
Chicago, Milwaukee & St. Paul.....	5,477	12.6
Southern Pacific.....	4,285	16.6
Atchison, Topeka & Santa Fe.....	3,735	22.7
Erie.....	2,823	25.1
Missouri Pacific.....	2,333	24.0
Pennsylvania.....	1,907	6.1
Louisville & Nashville.....	1,542	13.2
New York Central.....	1,430	10.0
Chesapeake & Ohio.....	1,182	19.1
Canadian Pacific.....	1,005	12.1
Averages for the group.....		18.2
New York, Ontario & Western.....	877	18.2
Western Union.....	737	9.2
Norfolk & Western.....	617	26.2
Missouri, Kansas & Texas.....	603	26.0
Texas & Pacific.....	599	25.9
Chicago & Northwestern.....	463	10.3
Illinois Central.....	351	7.6
Pacific Mail.....	316	26.5
Delaware & Hudson.....	299	8.7
Wabash.....	279	28.2
Cleveland, Cincinnati, Chicago & St. Louis.....	279	18.8
Delaware, Lackawanna & Western.....	236	11.2
Wheeling & Lake Erie.....	227	39.9
Denver & Rio Grande.....	220	27.4
Central of New Jersey.....	219	16.8
Averages for the group.....		19.8
Wisconsin Central.....	159	45.0
Chicago, St. Paul, Minneapolis, & Omaha.....	148	16.6
Canada Southern.....	132	7.7
Iowa Central.....	89	26.9
New York, Chicago & St. Louis.....	88	23.2
Pittsburgh, Cincinnati, Chicago & St. Louis.....	84	23.2
New York, New Haven & Hartford.....	81	9.8
Lake Erie & Western.....	73	27.2
Minneapolis & St. Louis.....	70	38.7
Duluth, South Shore & Atlantic.....	42	31.3
United States Express.....	15	16.6
American Express.....	12	10.5
Wells Fargo.....	6	13.9
Averages for the group.....		22.4

NOTE.—The number of shares sold has been divided by two in case of the stocks having a par value of \$50.

maker of index numbers in proportion as they rest upon factors which affect the prices of stocks. For instance, it is necessary to have all the geographical groups represented only so far as there is a characteristic difference in the pace at which railway traffic grows in the East and West, the North and South; or in the way in which railway finances are managed in different sections; or in the policy of different state commissions toward railways, etc. (2) In so far as the groups do rest upon factors which affect prices they overlap so much that the stocks within a given group usually present wide contrasts of behavior. For example, in the Pacific group we find Union Pacific beside Denver & Rio Grande; in the southern group Norfolk & Western beside Missouri Pacific, etc. (compare Tables VIII and X). (3) Many of the groups are unstable: the price-controlling factors which lend them significance have little in common with the physical conditions of supply which mark off, say, farm crops from mineral products. Stocks come into and pass out of the dividend-paying class; the active stocks of one year are inactive later; the "Harriman stocks" are not a "natural kind," etc. Indeed, new groupings may arise at any time to eclipse the older ones in importance, as the "war-stocks" appeared in 1914-15.

Since classification of stocks on the basis of factors affecting fluctuations in price is more complicated and unstable than the similar classification of commodities, one must be content with less definite rules of procedure. One can hardly say more than this: So far as your means allow, in each investigation include all the stocks for which you can find satisfactory quotations; arrange them in all the ways that appear significant for your purpose and find what peculiarities of fluctuation each group exhibits; make clear to yourself and to others what part of the field your groups and your totals cover and what parts they omit.

#### X. WEIGHTING

So long as statisticians expected but rough results from their index numbers of commodity prices at wholesale, they treated systematic weighting as a theoretical refinement in method which made little difference in the results. What pleased them was to

find that their simple and weighted averages showed the same general trend. But as experience has demonstrated that under favorable circumstances the margin of uncertainty in such work may be reduced to less than, say, 10 per cent of the results, makers of commodity index numbers have begun to regard proper weighting as practically important. Is it important also in making index numbers of stock prices?

Hitherto most stock indexes have been "simple" averages of actual or relative prices.<sup>1</sup> Now simple averages are not averages into which no weights enter, or in which all stocks have the same weights; they are really averages in which the weights have not been systematically planned but left to chance. What degree of influence any stock in a given sample will exercise upon the results in a simple series depends both upon the original quotations and upon the way in which they are worked up. For example, an arithmetic mean of actual prices in effect assigns heavy weights to the stocks that command high prices per share and light weights to stocks that are cheap. But if these actual prices are turned into relatives and the arithmetic means are made from the latter figures, the weighting is likely to be revolutionized. For now the influence of a given stock depends on a radically different factor, not on its price in dollars and cents as compared with the prices of other stocks in the sample, but on the percentage which the price on the date in question bears to the price of the same stock in the period chosen as base as compared with the corresponding percentages for the other stocks. A shift to a new base commonly alters the relative magnitude of these percentages and therefore changes the weights once more. Finally, the substitution of geometric means or medians for arithmetic means gives an entirely new twist to the whole situation. In a geometric mean the influence of a stock depends upon the comparative magnitude of the *ratios* of change

<sup>1</sup> The few weighted series have been averages of all stocks sold on a given exchange (for example, Table I, series 8), or they have been aggregates of the amounts of the several securities outstanding, multiplied by their current prices. In both cases the weights have been subject to alteration. Hence the weighted series have shown the combined results of *two* sets of changes, and therefore have not been clear-cut measures of fluctuations in prices. The confusion is particularly serious in the series from the *Financial Review*, given in Table I.

which its price undergoes, and it matters not whether actual or relative prices are used or on what base the relative prices are computed, for none of these matters affect the ratios of change, which alone count. In a median it does make a difference whether actual or relative prices are averaged and on what base the relatives are computed; but the influence which any stock exercises upon the result depends solely on whether its actual or relative price happens to be at, above, or below the middle of the whole series after the data have been arranged in numerical order. The magnitude of its deviation from the middle position has no effect.

Since all index numbers are really weighted, the only question is whether these weights should be tacit or avowed, obscure or clear, left to chance or controlled on some intelligible principle. This question is one of great moment, particularly when one is dealing with stocks. For we found in Section V that different schemes of systematic weighting produce large differences in results, when the weights themselves differ notably. We also found that the different schemes of haphazard weighting tacitly introduced by changing from averages of actual prices to averages of relatives, or by shifting the base on which relatives are computed, cause wide divergences. Finally, in most cases the series with systematic weights and the series with haphazard weights differ from each other at least as much as they differ among themselves. If systematic weighting is desirable in making commodity indexes where it leads to comparatively moderate differences in results, a fortiori it is desirable in making stock indexes where the differences produced in results are much wider.

Few men would hesitate to say that the price of Pennsylvania stock is more important than the price of Duluth, South Shore & Atlantic stock and deserves to have more weight in an index number. It is more important because there is more Pennsylvania stock in the hands of investors, individual and corporate; because the Pennsylvania does much the bigger business; because Pennsylvania stock is a more important article of commerce—more of it changes hands year by year.

These three reasons imply three different criteria of the importance of a given stock, criteria upon which may be based three sets

of weights, each of which is appropriate for special ends. If the aim is to show the average changes in the prices of securities held by the public, the amount of stock outstanding yields the logical set of weights. If the aim is to throw light on the changes in the prices of business enterprises as such, then gross earnings, the best available gauge of volume of business transacted, may be used as weights. If the aim is to find average changes in the prices of stocks that are traded in, then the number of shares sold should be used. Other aims might make still other systems of weights desirable. The three enumerated are all that I have worked out. How they compare with one another is shown by columns 4, 5, and 6 of Table XXI.<sup>1</sup>

But to what ought these weights be applied—to actual prices or to relative prices worked out on some chosen base? That is equivalent to the question: What weights ought to be used on the actual prices? For any average of relative prices is itself a weighted average of actual prices in disguise. For example, the preceding index numbers made by averaging relative prices on the 1890-99 base are equivalent to averages of actual prices weighted by the factors required to make the average actual price of each stock in that decade equal 100. What these factors are, after reduction to the scale of 100.0 for the sake of comparison with the other systems of weights, is shown in column 3 of Table XXI.<sup>2</sup>

<sup>1</sup> In all three systems arithmetic means for the twenty-five years 1890-1914 have been used. A high degree of accuracy cannot be claimed for the original data, which were compiled from the supplements of the *Commercial and Financial Chronicle* and the *Financial Review*, eked out by the use of railway and stock-exchange manuals. Probably the statistics of shares sold are the most accurate on the whole, though a few interpolations were necessary here, e.g., to supply data for Pennsylvania in 1890-96. The amount of stock outstanding in a given year is not always reported alike in two different sources, and statistics of earnings are often confused by changes in the fiscal year, changes in methods of accounting, etc. In particular the earnings of express companies in the years before the Interstate Commerce Commission was empowered to require reports from them had to be interpolated on the assumption that in earlier years they bore the same ratio to the earnings of the railways that they have borne since. But after all allowances are made, it is probable that each of these three sets of weights is a closer approximation to what the figures purport to show than can be reached in making weights for a large index number of commodity prices at wholesale.

<sup>2</sup> For example, the average price of Chesapeake & Ohio stock in 1890-99 was \$20.81, and its average price in 1915 was \$47.70. Its relative price in 1915 on the



TABLE XXI

DATA CONCERNING THE WEIGHTED INDEX NUMBERS OF THE ACTUAL PRICES OF COMMON STOCKS IN FORTY TRANSPORTATION COMPANIES

	AVERAGE PRICES IN 1890-1915		WEIGHTS IMPLICIT IN AVERAGES OF RELATIVE PRICES COMPUTED ON THE BASIS OF THE 1890-1915 PERIOD	WEIGHTS BASED UPON AVERAGE			PERCENTAGES OF THE TOTALS MADE FROM AVERAGE ACTUAL PRICES IN 1890-1915 MULTIPLIED BY WEIGHTS			
	In Dollars and Cents (1)	In Percentages of the Total (2)		Amount of Stock Outstanding 1890-1914 (4)	Earnings of Common Stocks 1890-1914 (5)	Number of Shares Sold 1890-1914 (6)	Implicit in 1890-99 Relatives (7)	Based on Stock Outstanding (8)	Based on Earnings (9)	Based on Shares Sold (10)
Total, forty stocks.....	\$3,227.50	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
North Atlantic railways.....	470.83	14.6	82	27.8	24.4	13.7	8.8	32.3	27.6	12.0
New York, New Haven & Hartford	178.82	5.5	3	3.5	3.7	0.1	1.3	7.0	6.7	0.2
New York, Ontario & Western.....	28.71	0.9	34	2.5	0.5	1.7	2.3	0.8	0.1	0.6
New York Central.....	116.24	3.6	6	6.3	6.4	2.8	1.7	8.2	7.5	4.0
Pennsylvania.....	121.24	3.8	5	11.0	10.0	3.7	1.4	15.0	12.3	5.5
Erie.....	25.82	0.8	34	4.5	3.8	5.4	2.1	1.3	1.0	1.7
Anthracite coal railways.....	722.04	22.4	38	6.6	8.3	14.7	11.7	11.6	15.0	16.4
Central Railroad of New Jersey.....	181.13	5.6	6	1.1	1.7	0.4	2.6	2.3	3.1	0.9
Delaware & Hudson.....	151.52	4.7	5	1.6	1.3	0.6	1.8	2.7	2.0	1.1
Delaware, Lackawanna & Western.....	310.63	9.6	4	1.2	2.4	0.4	3.0	4.2	7.6	1.5
Reading.....	78.76	2.5	23	2.7	2.9	13.3	4.3	2.4	2.3	12.9
Middle-western railways.....	385.01	11.9	232	10.2	11.5	2.8	16.9	7.7	7.9	2.0
Cleveland, Cincinnati, Chicago & St. Louis.....	61.00	1.9	13	1.5	1.8	0.5	1.9	1.0	1.1	0.3
Wheeling & Lake Erie.....	13.26	0.4	36	0.7	0.4	0.4	1.1	0.1	0.1	0.1
New York, Chicago & St. Louis.....	33.14	1.0	41	0.6	0.7	0.2	3.3	0.2	0.2	0.1
Wabash.....	12.26	0.4	68	1.6	1.8	0.5	2.0	0.2	0.2	0.1
Pittsburgh, Cincinnati, Chicago & St. Louis.....	59.29	1.8	24	1.2	2.2	0.1	3.4	0.8	1.3	0.1

Canada Southern.....	60.42	1.9	11	0.7	0.6	0.3	1.6	0.5	0.4	0.2
Lake Erie & Western.....	23.14	0.7	33	0.5	0.4	0.1	1.8	0.1	0.1	0.1
Illinois Central.....	122.50	3.8	6	3.4	3.6	0.7	1.8	4.8	4.5	1.0
Northwestern railways.....	456.94	14.2	282	8.7	11.4	12.4	18.1	10.3	13.2	16.9
Chicago, Milwaukee & St. Paul.....	114.05	3.5	8	3.0	4.4	10.5	2.2	3.8	5.0	14.6
Chicago & Northwestern.....	147.50	4.6	5	3.0	4.5	0.9	1.8	5.0	6.7	1.5
Chicago, St. Paul, Minneapolis & Omaha.....	107.12	3.3	12	0.8	1.0	0.3	3.1	1.0	1.1	0.4
Duluth, South Shore & Atlantic.....	9.06	0.3	102	0.5	0.3	0.1	2.2	0.1	0.1	0.1
Iowa Central.....	16.77	0.5	65	0.4	0.3	0.2	2.6	0.1	0.1	0.1
Minneapolis & St. Louis.....	38.14	1.2	31	0.3	0.3	0.1	2.8	0.1	0.1	0.1
Wisconsin Central.....	24.30	0.8	59	0.7	0.6	0.3	3.4	0.2	0.1	0.1
Southern railways.....	297.71	9.2	198	14.5	12.6	13.3	18.7	8.4	7.4	9.4
Chesapeake & Ohio.....	41.16	1.3	29	2.6	1.7	2.3	2.9	1.2	0.7	1.2
Norfolk & Western.....	55.47	1.7	50	2.4	1.9	1.2	6.6	1.5	1.1	0.8
Louisville & Nashville.....	102.01	3.1	10	2.5	3.0	3.0	2.4	2.9	3.1	3.7
Missouri Pacific.....	56.34	1.7	14	2.9	3.4	4.5	1.9	1.8	1.9	3.1
Missouri, Kansas & Texas.....	21.35	0.7	46	2.4	1.6	1.2	2.4	0.6	0.4	0.3
Texas & Pacific.....	21.38	0.7	49	1.7	1.0	1.1	2.5	0.4	0.2	0.3
Pacific railways.....	371.28	11.5	119	25.5	23.8	40.8	16.7	22.9	19.1	41.2
Atchison, Topeka & Santa Fe.....	62.57	1.9	28	5.2	5.7	7.2	4.2	3.7	3.6	5.5
Denver & Rio Grande.....	23.31	0.7	40	1.6	1.3	0.4	2.2	0.4	0.3	0.1
Southern Pacific.....	60.96	1.9	23	8.1	7.7	8.3	3.3	5.6	4.8	6.2
Union Pacific.....	93.74	2.9	20	5.7	4.4	23.0	3.3	6.0	4.2	26.4
Canadian Pacific.....	130.70	4.1	8	4.9	4.7	1.9	2.5	7.2	6.2	3.0
Express companies, etc.....	523.69	16.2	49	6.7	8.0	2.3	9.1	6.8	9.8	2.1
American Express.....	165.39	5.1	5	0.8	2.3	0.1	2.0	1.5	3.8	0.2
United States Express.....	76.52	2.4	12	0.4	1.1	0.1	2.2	0.3	0.9	0.1
Wells Fargo.....	168.67	5.2	5	0.4	1.7	0.1	2.0	0.8	2.9	0.2
Pacific Mail Steamship.....	31.51	1.0	20	0.9	0.4	0.6	1.5	0.3	0.1	0.2
Western Union.....	81.60	2.5	7	4.2	2.5	1.4	1.4	3.9	2.1	1.4

Similarly, the index numbers of relative prices on the preceding-year base are averages of actual prices, each weighted by the multiplier, which makes its price in the year before equal 100.<sup>1</sup>

In weighting relative prices, then, we are weighting already-weighted actual prices. Upon the final result, therefore, each stock will have an influence proportioned, not to its figure in the formal scale of weights, but to this figure combined with its actual-price-times-another-weight. Likewise, in weighting actual prices themselves we give each stock an influence upon the result which depends, not simply on the weight, but upon the product of the weight times the price. Columns 7-10 in the table show roughly how much these "net influences" differ from the weights in columns 3-6.<sup>2</sup>

The first step in weighting, therefore, should be to decide what proportionate influence we wish each stock to exercise upon the final results. Of course that depends upon the end in view. For example, in measuring the changes in the market value of stocks held by investors the importance of each of our sample stocks depends both on the amount in the hands of investors and on the actual price. Weights based on amounts outstanding should therefore be applied to actual prices. If for some other purpose

1890-99 base was therefore  $100 (47.70 \div 20.81) = 229\pm$ . But this equation may be written  $47.70 (100 \div 20.81) = 229\pm$ ; or  $47.70 \times 4.81 = 229\pm$ . The sums of the factors required to make the 1890-99 prices of each of the forty stocks = 100 is 166.63. The factor for Chesapeake & Ohio is 2.9 per cent of that sum—which is the figure for this stock entered in column 3 of Table XXI. Multiply the actual price of each stock in any year by its entry in this column, cast up the totals, divide by the sum of these multipliers times 1890-99 prices (\$24,190.80), point off the proper number of places, and you have the arithmetic means of relative prices shown in Table III, column 2, barring slight discrepancies caused by the dropping of different fractions in the two computations.

<sup>1</sup> In computing an index number it is mainly a question of convenience whether to make relative prices by dividing by the base price and multiplying by 100, or to make the same figures by multiplying by 100 divided by the base price and to call the results weighted aggregates of actual prices. The latter course, however, has the advantage of keeping the weights used clearly before the mind and of showing just how the relative-price series differ from the original actual-price series.

<sup>2</sup> To show accurately the average net influence exercised upon the results by each stock, it would be necessary to carry through a computation for each year separately. But the present figures, based upon average actual prices for the whole period covered, serve well enough for the present illustrative purpose.

we think that the fluctuations of each stock should have an influence proportionate simply to the gross earnings of each corporation, then we should not apply weights based upon earnings directly to actual prices, but should first make the average actual prices of all the stocks the same for the period covered by applying one set of equalizing weights, and then multiply these equated prices by the weights based upon earnings. In this case, however, it would be quicker to begin by making the two sets of weights into one, and then to multiply the actual prices of the stocks by the consolidated weights.

In the last resort, then, the other than formal differences among the several index numbers shown in Tables I, II, and III resolve themselves into differences in the stocks included, in the methods of averaging, and in the methods of weighting. Any reader who so desires may eliminate the first two causes of differences by confining his attention to the arithmetic means of forty common stocks and then trace the peculiarities of each series back to their source by noting which stocks each series weights most heavily according to Table XXI and how these stocks have fluctuated according to Tables VIII-X. The only difficulty here is that the weighting of actual prices implicit in the percentages of change in price from one year to the next cannot be shown in a single column, because these weights vary from year to year.

#### XI. CONCLUSIONS

It was said in Section II, and the remark has been repeated incidentally several times, that the choice of methods in making an index number of stocks should be guided by the specific purpose in view. It follows that the index number that is best for any purpose depends upon the specific phase of price fluctuations which that purpose requires to be measured.

Strictly interpreted, this obvious but often-neglected rule bars out the question: What is the best index number at large? Perhaps there is no single series among all the stock index numbers presented above that is not "the best" for some imaginable use. But, by way of conclusion, we may point out what fluctuations in the prices of stocks can be measured with the narrowest margin of error, and argue that the index number which best represents these

most measurable fluctuations is the best "general-purpose" series; the index number to be recommended for use by the general reader, and by the specialist also, when his particular aim does not definitely demand some differently constructed series, in spite of its inferior accuracy.

Along this line a confident opinion can be given. Geometric means of the ratios of change in quotations within brief periods, such as from one year to the next, have been shown to be the most accurate measures of fluctuations in the prices of stocks. That is, Table II, column 3, gives the most trustworthy among the preceding index numbers. As a "general-purpose" series, however, this index number could be improved (1) by including preferred as well as common stocks, (2) by adding securities of other than transportation companies if quotations could be found, (3) by using geometric instead of arithmetic means in striking the average annual prices from the monthly figures, (4) by weighting the figures for each stock in accordance with average earnings or amount of stocks outstanding, and (5) by using geometric means in computing these weights.

For measuring fluctuations covering longer periods of time geometric means are again the most representative averages. But the farther apart grow the years between which price comparisons are made the less accurate grow the results obtained from a given body of quotations and the smaller grows the list of stocks for which continuous series of quotations can be had. It is true that the successive percentages of change in price from one year to the next can be multiplied into each other to make a continuous "chain index"; but, while each link has a narrow margin of error, the errors are cumulative, so that a comparison between the two ends of the chain becomes less trustworthy the longer the chain is made.<sup>1</sup> Of course the same difficulty inheres in the relative

<sup>1</sup> Geometric means are particularly well adapted to the making of such chain indexes. Say that in a chain series 1890 has an index of 100 and 1915 one of 125. The 25 per cent gain represents a quarter of the prices of 1890 or a fifth of the prices of 1915, though twenty-four separate ratios to prices in other years have been used in passing from 1890 to 1915. But if arithmetic means have been used, the meaning of the figure for 1915 in terms of prices in 1890 (or in 1915) has been hopelessly obscured. Compare "The Making and Using of Index Numbers," p. 38.

prices on a fixed base that may be made from the geometric means of actual prices. No refinement of methods can mend the fundamental defect of the data: the ratios of change in stock prices between years far apart are so widely and irregularly scattered that no average made from them can have a high representative value.<sup>1</sup>

The best way to diminish, since we cannot remove, this difficulty is to break the long periods up into parts, to compute fresh index numbers for each part, and to string these index numbers together. The advantages of this shift are (1) that a larger "sample" of stocks with continuous quotations can be had for short periods, and (2) that the fixed-base relatives will show a less irregular distribution. Pushed to extremes, this course would lead to the making of a geometric-mean index number of all stocks quoted both in 1890 and in 1891, of a second index number of all stocks quoted both in 1891 and in 1892, and so on to date. The main defect of such a series, after the yearly percentages had been linked together in a chain index, would be that no one could be sure what part of the fluctuations shown was due to change in prices and what part to changes in the stocks quoted. Hence price comparisons between 1890 and 1915 would still be dubious. Perhaps a middle course is the least objectionable: Make a new index number from a new sample of stocks every ten or twenty years, using geometric means; each time that a new series is made compute overlapping figures for a few years both from the old and from the new samples; find what part of the changes in those years is due to alterations in the list of stocks, and, finally, allow for these differences as well as may be in joining the two index numbers together. The price comparisons that could be extended in this way over long periods of time would not indeed possess the accuracy of our year-to-year figures, but they would be more trustworthy than any of the fixed-base series given above.

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<sup>1</sup> Compare Table XVIII.